The role of proton-proton collisions in the turbulent solar wind by means of hybrid Boltzmann-Maxwell simulations

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Abstract

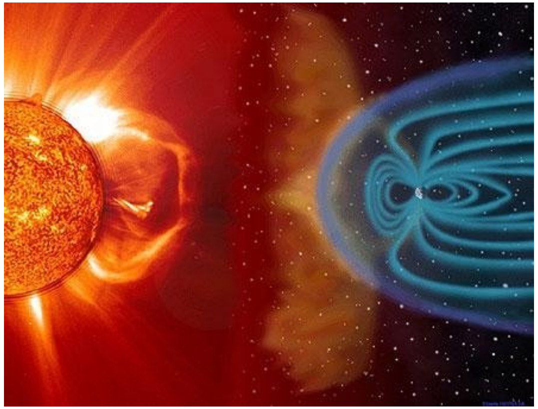
- **Collisions in collisionless plasmas**
  - Heating and dissipation in weakly collisional plasmas: ***what is the role of collisions?***

- **Collisionality enhancement due to small-scale velocity-space structures**
  - Existence of several characteristic times associated with fine structure dissipation[1]
  - Effects of nonlinear vs linearized collisional operator[2]

- **Collisions in a self-consistent hybrid plasma simulations**
  - Inhibition of non-Maxwellian features[3]

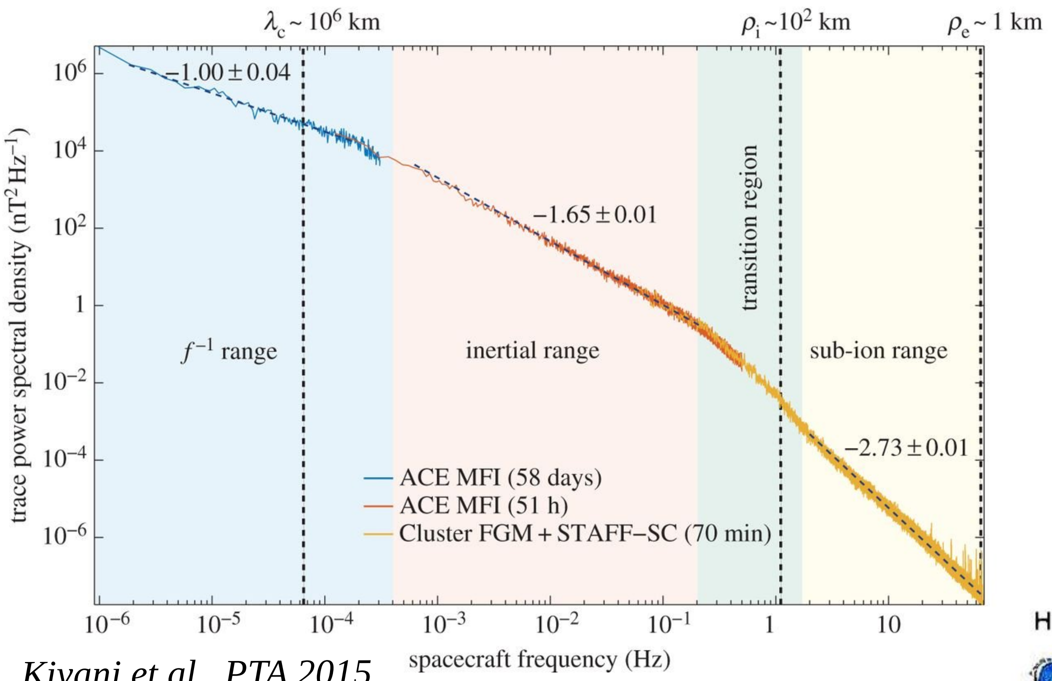
- **Conclusions**

[1] ***Pezzi***, *Valentini and Veltri, PRL* ***116***, *145001 (2016)*
[2] ***Pezzi***, *JPP* ***83***, *555830301 (2017)*
[3] ***Pezzi*** *et al., under review (2019)*



M16 eagle nebula.
NASA/Hubble

The plasma heating problem

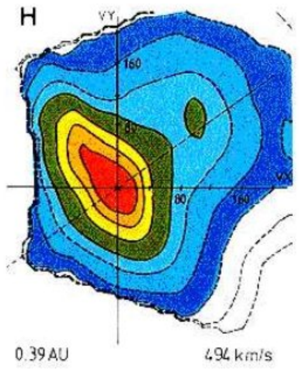


Kiyani et al., PTA 2015

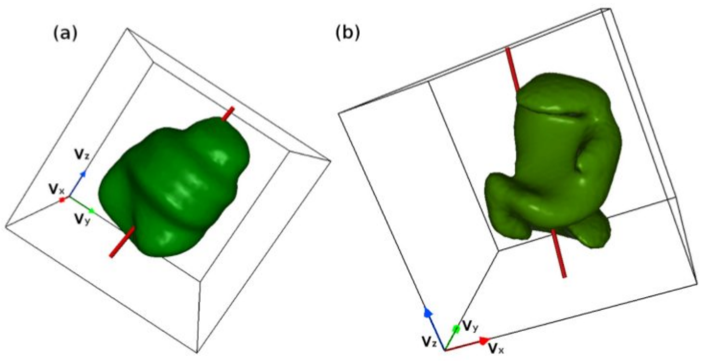
Turbulence

Kinetic physics

Starting from the sub-ion range, **a kinetic approach is mandatory to describe the plasma dynamics!**



Marsch et al., JGR 1982



Valentini et al., NJP 2016

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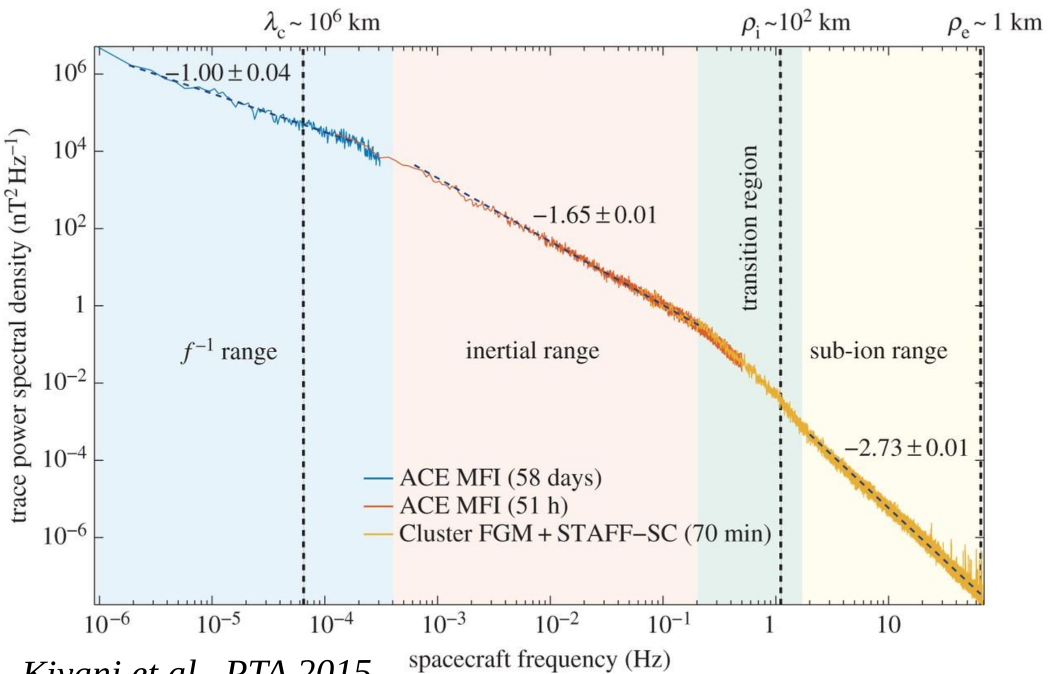
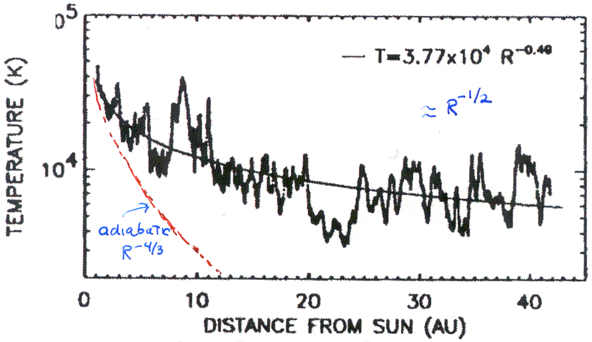


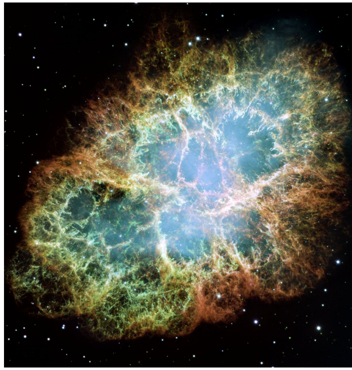
Kiyani et al., PTA 2015

Turbulence → Kinetic physics → Heating

Several *local* heating mechanisms, often based on the ***collisionless assumption***, have been proposed but a definitive answer is still missing.



Richardson et al., GRL 1995



Crab Nebula NASA/Hubble

Collisions in almost collisionless plasmas

… Collisions, although they are usually neglected in studying a weakly collisional plasma, are the decisive effect to introduce irreversibility in the system!

Quasi-Maxwellian approach

Frequency	Slow wind	Fast wind
proton cyclotron	~ 0.1 Hz	~ 0.1 Hz
electron cyclotron	$\sim 2 \times 10^2$ Hz	$\sim 2 \times 10^2$ Hz
plasma	$\sim 2 \times 10^5$ Hz	$\sim 1 \times 10^5$ Hz
proton-proton collision	$\sim 2 \times 10^{-6}$ Hz	$\sim 1 \times 10^{-7}$ Hz

Bruno&Carbone, LRSP, 2013

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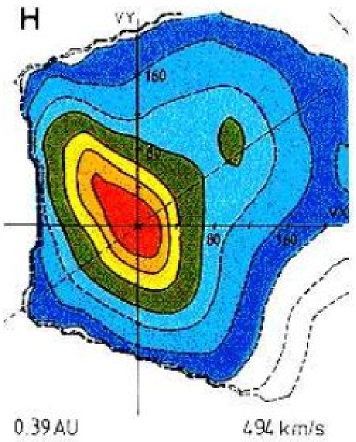
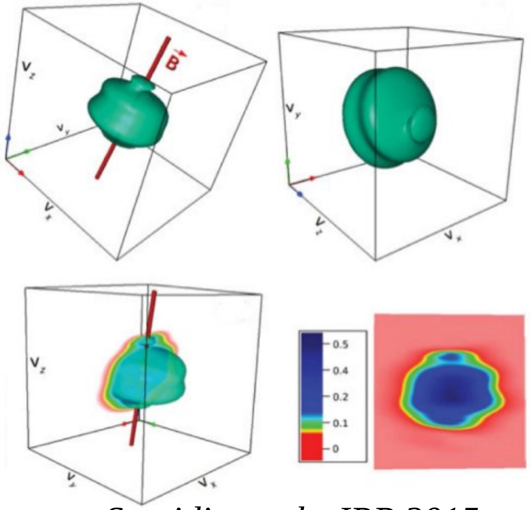
Bruno&Carbone, LRSP, 2013

Is SW quasi-Maxwellian? NO!

Proton VDFs at kinetic scales display strong distortions due to wave-particle interactions and turbulence



Marsch et al., JGR 1982



Servidio et al., JPP 2015

Modeling inter-particle collisions

Vlasov – Maxwell system + Collisional operator

$$\left[\frac{\partial}{\partial t} + \mathbf{v}\cdot\frac{\partial}{\partial \mathbf{r}} + (\mathbf{E} + \mathbf{v}\times\mathbf{B})\cdot\frac{\partial}{\partial \mathbf{v}}\right] f(\mathbf{r},\mathbf{v},t) = \left.\frac{\partial f}{\partial t}\right|_{coll}$$

$$\nabla\cdot\mathbf{E} = \rho$$

$$\nabla\cdot\mathbf{B} = 0$$

$$\nabla\times\mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

$$\nabla\times\mathbf{B} = \frac{\partial \mathbf{E}}{\partial t} + \mathbf{j}$$

$$\left.\frac{\partial f}{\partial t}\right|_{coll} = \nu_0 \frac{\partial}{\partial v_i}\int d^3v' U_{ij}(\mathbf{u})\left[f(\mathbf{v}')\frac{\partial f(\mathbf{v})}{\partial v_j} - f(\mathbf{v})\frac{\partial f(\mathbf{v}')}{\partial v'_j}\right]$$

Velocity space gradients which may enhance collisional effects

$$U_{ij}(\mathbf{u}) = \frac{\delta_{ij}u^2 - u_i u_j}{u^3} \qquad \mathbf{u} = \mathbf{v} - \mathbf{v}'$$

Modeling inter-particle collisions

… due to the huge computational cost of the Landau collisional operator, we first focused on the **collisional relaxation of a spatially homogeneous force-free plasma ...**

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$$\nu_0 \simeq \nu_{SH}$$

Landau operator

N^6 for a 0D-3V grid!!

Numerical methods:

- 6th order FD scheme for velocity derivatives
- 4th order AB scheme for temporal evolution

$$\nu_{SH} \simeq 8 \times (0.714 \pi n e^4 \ \ln \ \Lambda)/(m^{0.5}(3 k_B T)^{3/2})$$

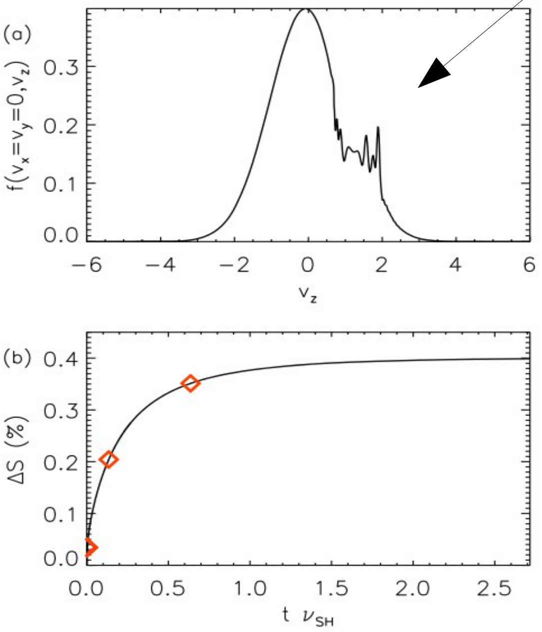
Normalization quantities: λ_D , $T=1/\upsilon_{SH}$,v_{th}

… Although strongly approximated, this system is useful to provide insights on the behavior of the ***kernel of the collisional operator***…

Existence of several characteristic times

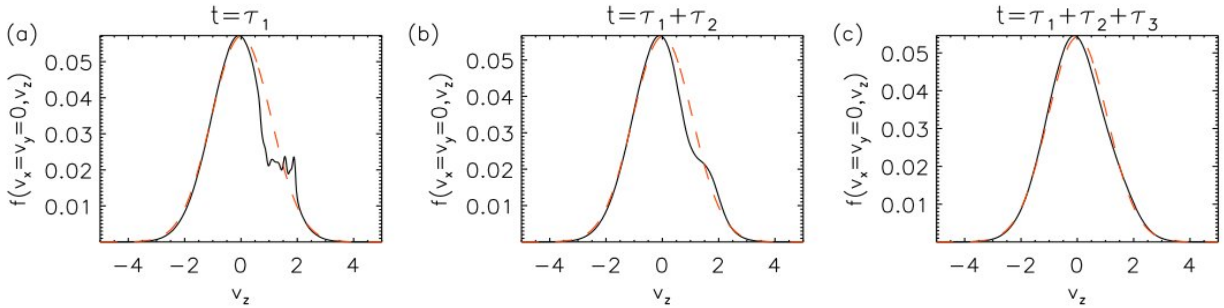
__Pezzi__, Valentini and Veltri, PRL __116__, 145001 (2016)

$$f(v_x, v_y, v_z) = f_{M,T_e}(v_x) f_{M,T_e}(v_y) \hat{f}_e(v_z)$$ → … No temperature anisotropy!



Characteristic times associated with fine velocity structures gets smaller as finer scales are present in the VDF

- $\tau_1 = 3.5 \cdot 10^{-3} \nu_{SH}^{-1} \rightarrow \Delta S_1 / \Delta S_{tot} = 13\%$ Spikes dissipation
- $\tau_2 = 1.3 \cdot 10^{-1} \nu_{SH}^{-1} \rightarrow \Delta S_2 / \Delta S_{tot} = 42\%$ Plateau smoothing
- $\tau_3 = 4.9 \cdot 10^{-1} \nu_{SH}^{-1} \rightarrow \Delta S_3 / \Delta S_{tot} = 40\%$ Final approach to equilibrium Maxwellian



Multi-exponential fit

$$\Delta S(t) = \sum_{i=1}^{K} \Delta S_i (1 - e^{-t/\tau_i})$$

Scenario confirmed in several numerical experiments...

Nonlinearities in the collisional operator

It is also interesting to extend the analysis by directly comparing the effects of the fully nonlinear Landau operator and its linearized version:

$$\frac{\partial f}{\partial t} = \left.\frac{\partial f}{\partial t}\right|_{coll} = \nu_0 \frac{\partial}{\partial v_i} \int d^3v' U_{ij}(\mathbf{u}) \left[f(\mathbf{v}') \frac{\partial f(\mathbf{v})}{\partial v_j} - f(\mathbf{v}) \frac{\partial f(\mathbf{v}')}{\partial v_j'} \right]$$

$$\frac{\partial f}{\partial t} = \left.\frac{\partial f}{\partial t}\right|_{coll} = \nu_0 \frac{\partial}{\partial v_i} \int d^3v' U_{ij}(\mathbf{u}) \left[f_0(\mathbf{v}') \frac{\partial f(\mathbf{v})}{\partial v_j} - f(\mathbf{v}) \frac{\partial f_0(\mathbf{v}')}{\partial v_j'} \right]$$

$$N_{v_x} = N_{v_y} = 51$$

$$N_{v_z} = 1601$$

$$U_{ij}(\mathbf{u}) = \frac{\delta_{ij} u^2 - u_i u_j}{u^3} \qquad \mathbf{u} = \mathbf{v} - \mathbf{v}'$$

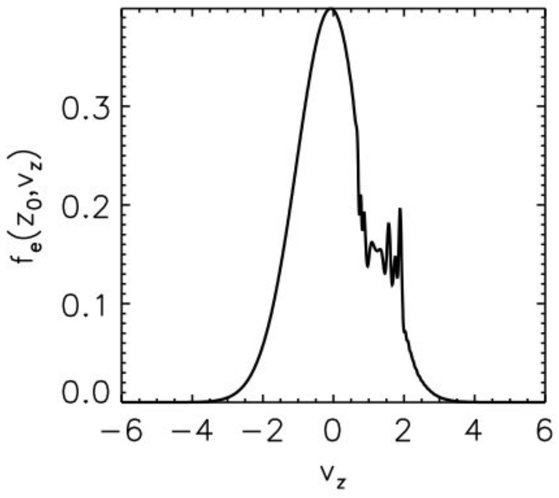
$$\nu_0 \simeq \nu_{SH}$$

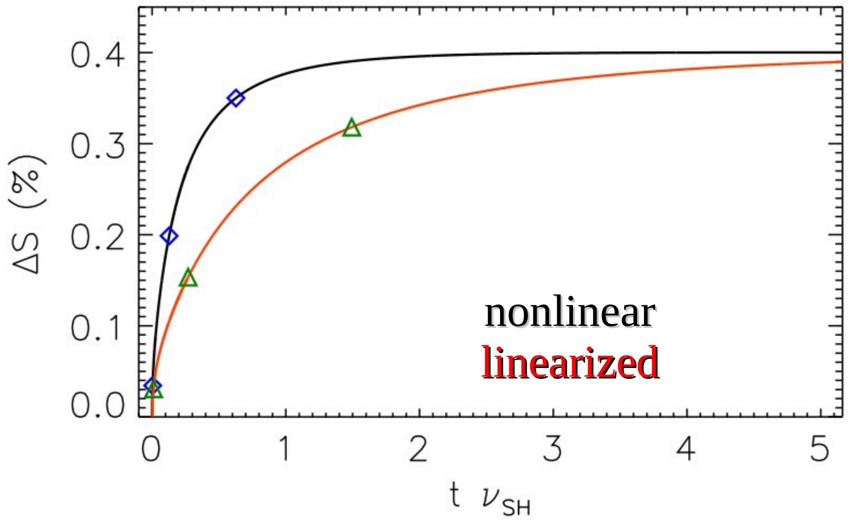
Coefficients (proportional to **v'**) of the Fokker-Planck like structures of the Landau operator have been linearized by introducing the Maxwellian distribution function f_0 associated with f

Pezzi, *JPP **83**, 555830301 (2017)*

Nonlinearities in the collisional operator

$f(v_x, v_y, v_z) = f_{M,T_e}(v_x) f_{M,T_e}(v_y) \hat{f}_e(v_z)$





NONLINEAR

$$
\begin{aligned}
&\text{(i)} \quad \tau_1^{nl} = 3.5 \times 10^{-3}\, \nu_{SH}^{-1} \rightarrow \Delta S_1^{nl}/\Delta S_{tot} = 13\,\%, \\
&\text{(ii)} \quad \tau_2^{nl} = 1.3 \times 10^{-1}\, \nu_{SH}^{-1} \rightarrow \Delta S_2^{nl}/\Delta S_{tot} = 42\,\%, \\
&\text{(iii)} \quad \tau_3^{nl} = 4.9 \times 10^{-1}\, \nu_{SH}^{-1} \rightarrow \Delta S_3^{nl}/\Delta S_{tot} = 40\,\%.
\end{aligned}
$$

LINEARIZED

$$
\begin{aligned}
&\text{(i)} \quad \tau_1^{lin} = 1.1 \times 10^{-2}\, \nu_{SH}^{-1} \rightarrow \Delta S_1^{lin}/\Delta S_{tot} = 11\,\%, \\
&\text{(ii)} \quad \tau_2^{lin} = 2.7 \times 10^{-1}\, \nu_{SH}^{-1} \rightarrow \Delta S_2^{lin}/\Delta S_{tot} = 23\,\%, \\
&\text{(iii)} \quad \tau_3^{lin} = 1.5\, \nu_{SH}^{-1} \rightarrow \Delta S_3^{lin}/\Delta S_{tot} = 63\,\%.
\end{aligned}
$$

Characteristic times associated with the linearized operator are **systematically bigger** compared to the ones recovered for the fully nonlinear operator.

Pezzi, *JPP* ***83***, *555830301 (2017)*

Collisions in hybrid kinetic plasma

To perform **self-consistent Eulerian simulations**, we are forced to simplify the Landau collisional operator, by adopting the **non-linear Dougherty operator:**

$$\left.\frac{\partial f}{\partial t}\right|_{coll} = \nu_0 \frac{\partial}{\partial v_i} \int d^3v' U_{ij}(\mathbf{u}) \left[f(\mathbf{v}') \frac{\partial f(\mathbf{v})}{\partial v_j} - f(\mathbf{v}) \frac{\partial f(\mathbf{v}')}{\partial v'_j} \right]$$

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$$C(f,f) = \nu \frac{n}{T^{3/2}} \frac{\partial}{\partial v_j} \left[T \frac{\partial f}{\partial v_j} + (v - V)_j f \right]$$

Dougherty et al., POF 1964; POF 1967
Anderson&O'Neil, POP 2007

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$$\frac{\partial \mathbf{B}}{\partial t} = -\nabla \times \boldsymbol{E} = \nabla \times \left[\boldsymbol{u} \times \boldsymbol{B} - \frac{\boldsymbol{j} \times \boldsymbol{B}}{n} + \frac{\nabla P_e}{n} - \eta \boldsymbol{j}\right]$$

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Dougherty operator’s similarities to the Landau operator:
I. It preserves the total mass, momentum and energy.
II. It satisfies the H-theorem for the growth of the Gibbs-Boltzmann entropy.
III. Qualitatively and quantitatively similar relaxation towards the equilibrium, upon rescaling time by a constant factor in the Dougherty operator case (**Pezzi** et al., JPP 2015).

Dougherty operator’s differences from the Landau operator:
I. Different physical behavior (and hence scaling) in Hermite space (**Pezzi** et al., PPCF 2019).

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2.5D SIM

$N_x = N_y = 512$
$N_{vx} = N_{vy} = N_{vz} = 71$
B=2 - η=1e-3
ν={0,1e-3}
Large scale **δB**, **δu**
δB/B0 = 1/3

~1M CPU-hours
@MARCONI
Valentini et al., JCP 2007

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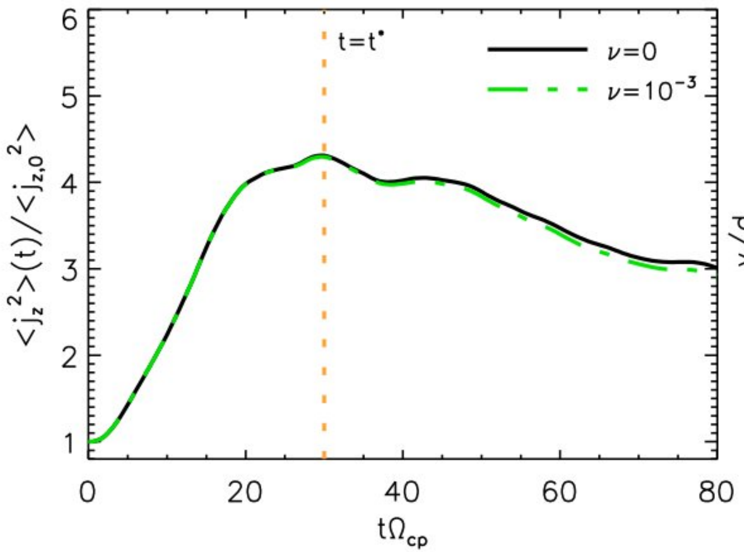
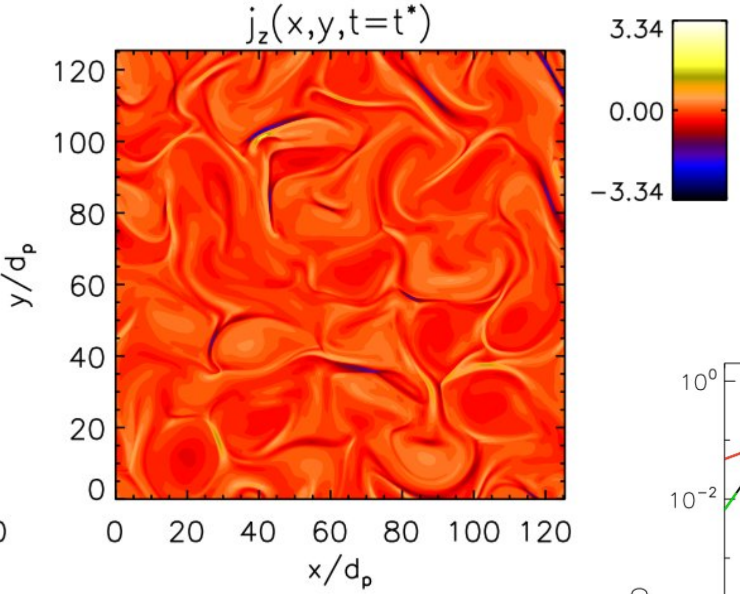
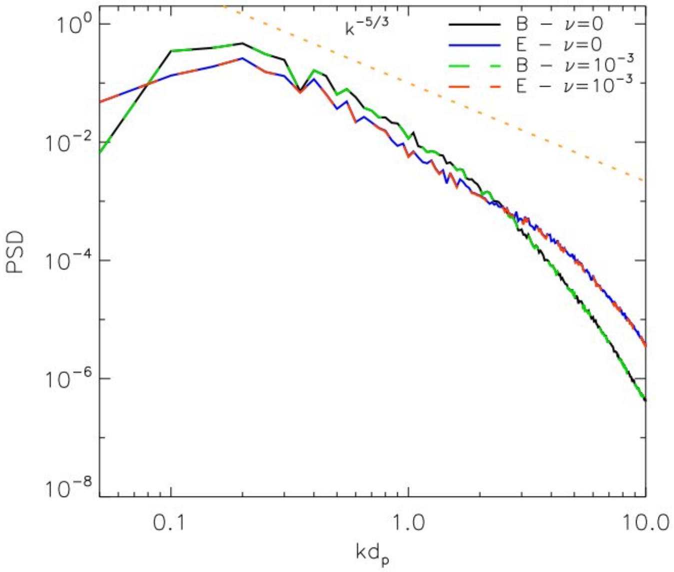






The statistical features of turbulence at proton scales are almost unaffected by the presence of the collisional operator...

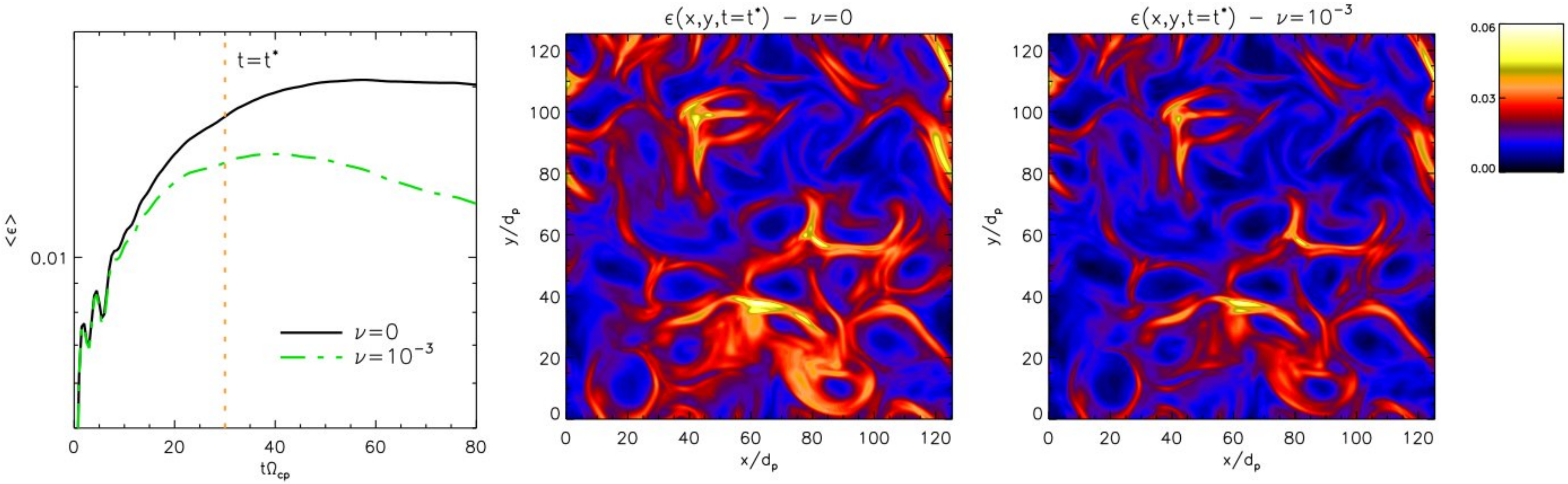
Pezzi *et al., under review (2019)*

Collisional effects on non-Maxwellian features

The turbulent cascade naturally generates non-Maxwellian features in the proton VDFs.
Collisions slowly but incessantly work to dissipate such structures.

$$\varepsilon(x,y,t) = \frac{1}{n}\sqrt{\int\left[f(\mathbf{x},\mathbf{u},t) - f_M(\mathbf{x},\mathbf{u},t)\right]^2 d^3\mathbf{u}}$$

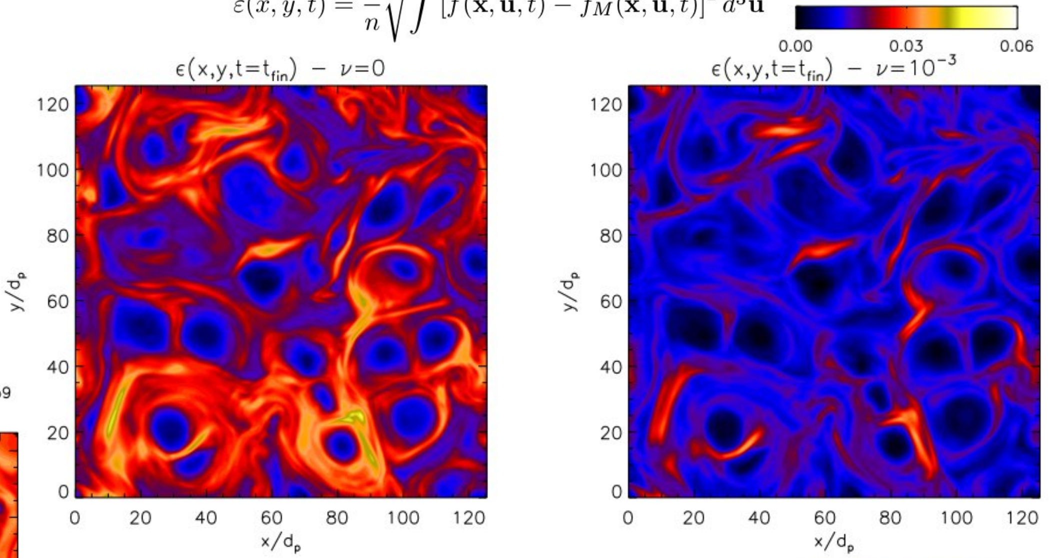
Greco et al., PRE 2012



Collisional effects on non-Maxwellian features

$t=t_{fin}$

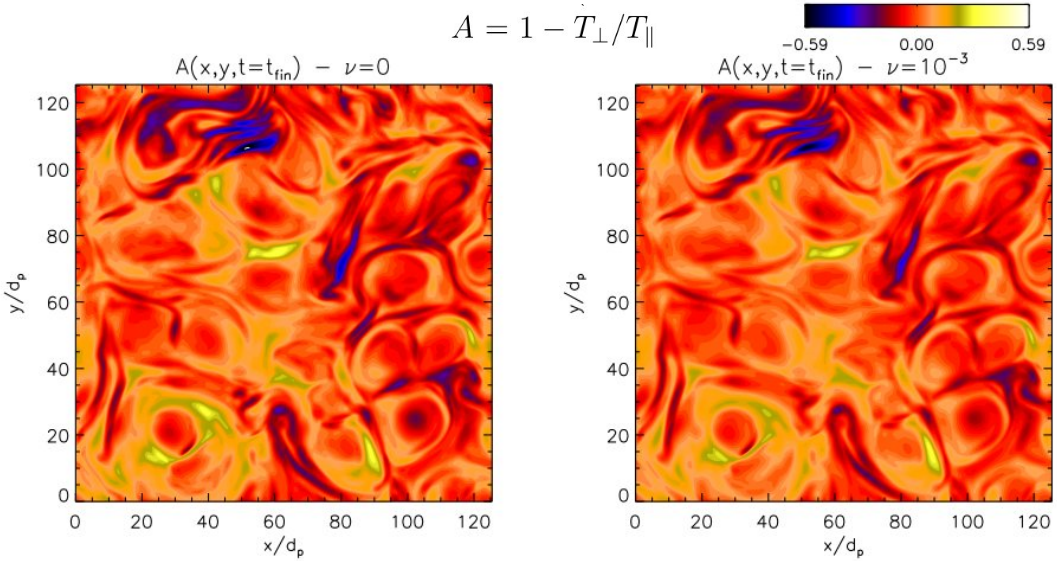
$$\varepsilon(x,y,t) = \frac{1}{n}\sqrt{\int \left[f(\mathbf{x},\mathbf{u},t) - f_M(\mathbf{x},\mathbf{u},t)\right]^2 d^3\mathbf{u}}$$



Temperature anisotropy is not affected by collisions, that **dissipate purely kinetic characteristics much faster than temperature anisotropies...**

... collisional enhancement due to fine velocity-space structures ?

$$A = 1 - T_\perp / T_\parallel$$



***Pezzi**, Valentini and Veltri, PRL **116**, 145001 (2016)*

Velocity-space Hermite decomposition

The velocity-space VDF is decomposed in the Hermite space:

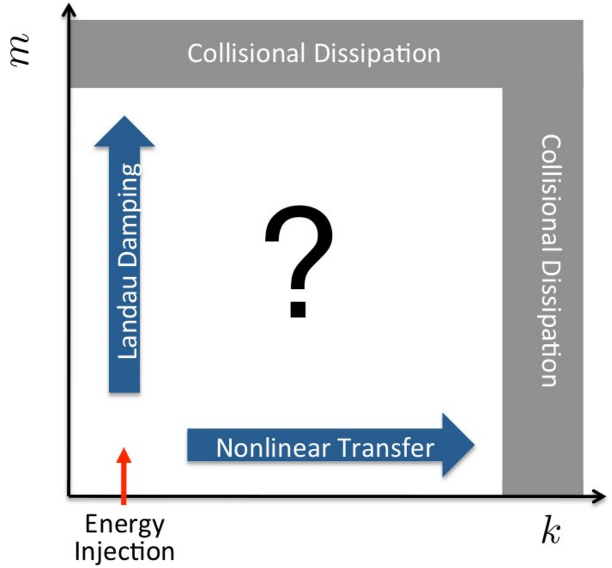
Hermite 3D decomposition: $f(\mathbf{v}) = \sum_{\mathbf{m}} f_{\mathbf{m}} \psi_{\mathbf{m}}(\mathbf{v})$

Hermite 3D coefficients $f_{\mathbf{m}} = \int_{-\infty}^{+\infty} f(\mathbf{v}) \psi_{\mathbf{m}}(\mathbf{v}) d^3 v$

Hermite polynomials:

$$\psi_m(v) = \frac{H_m\left(\frac{v-u}{v_{th}}\right)}{\sqrt{2^m m! \sqrt{\pi} v_{th}}} e^{-\frac{(v-u)^2}{2 v_{th}^2}} \longrightarrow \int_{-\infty}^{+\infty} \psi_m(v) \psi_l(v) dv = \delta_{ml}$$

$$H_m(v) = (-1)^m e^{v^2} \frac{d^m}{dv^m} e^{-v^2}$$



Plunk and Parker, EPJD 2014

Grad 1949, […] Schekochihin et al. JPP 2016, Servidio et al., PRL ***119****, 205101 (2017), Cerri et al. APJ 2018,* ***Pezzi*** *et al., POP* ***25****, 060704 (2018)*

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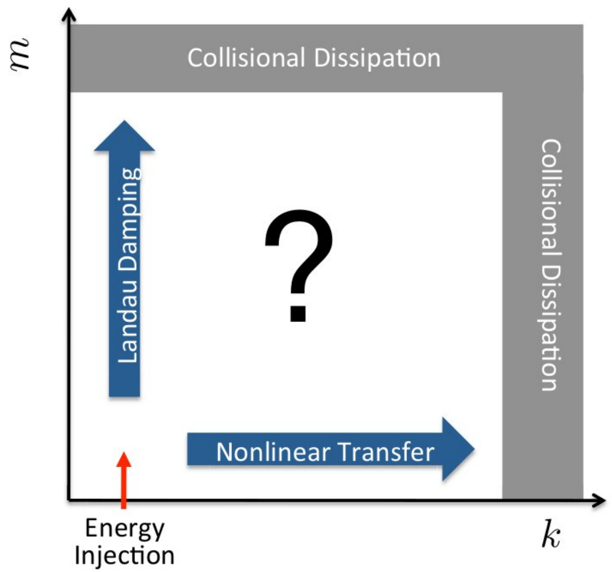
Grid opportunely shifted to avoid bulk speed (m~1) and temperature (m~2) fluctuations



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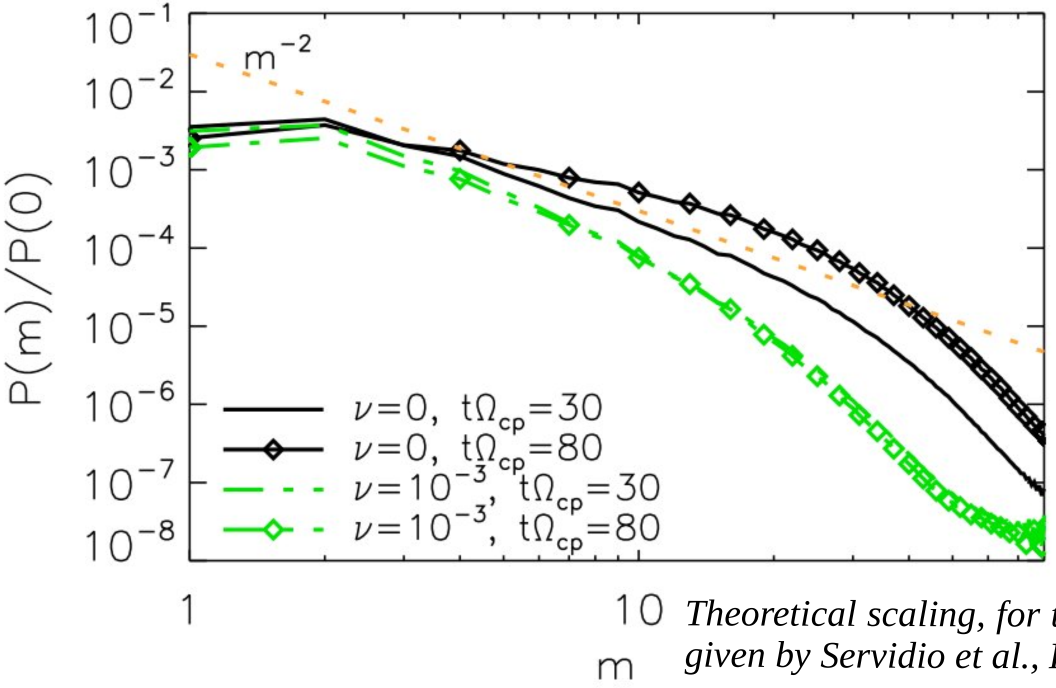
Omni-directional Hermite spectrum

- Spatial average on a 64x64 VDFs subset of the 512x512 VDFs set to ensure statistical convergence;
- Omni-directional spectrum obtained by summing on concentric *m*-shells

Collisions inhibit the development of the Hermite velocity-space cascade

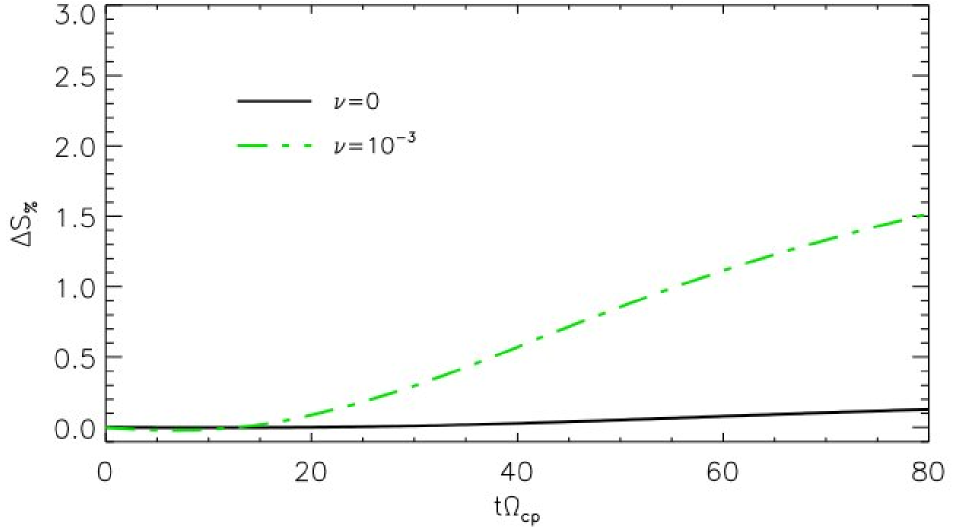


Theoretical scaling, for the collisionless case, given by Servidio et al., PRL, 2017

Entropy growth as a signature of dissipation

Proton-proton collisions → ***) Total energy is preserved (no heating as ΔT)**
***) Entropy increases!**

$$S(t) = -\int d^3r d^3v f \log f$$



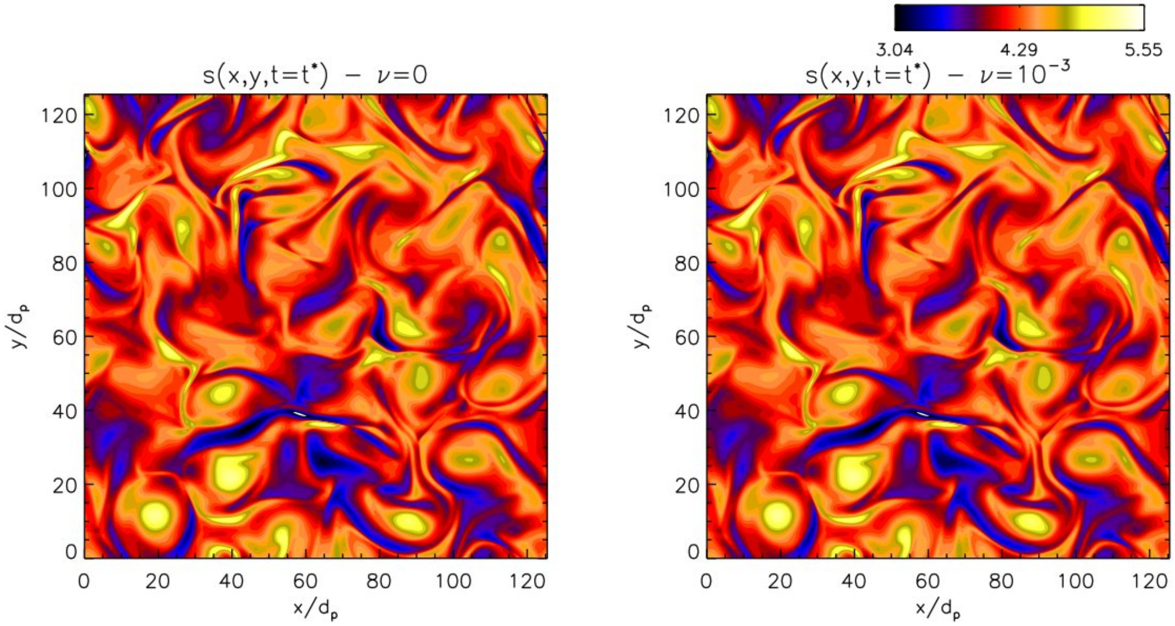
The **free-energy contained in the VDFs' non-Maxwellian structures** is, in principle, available in principle for the onset of several *collisionless processes*, such as *microinstabilities*…

… ***Collisions irreversibly dissipate*** it, generating the entropy growth...

Entropy vs Entropy density

$$S(t) = -\int d^3r d^3v f \log f \quad \dashrightarrow \quad s(\boldsymbol{x}, t) = -\int d^3v f \log f$$

Parks et al., 2012; Gary et al., 2018

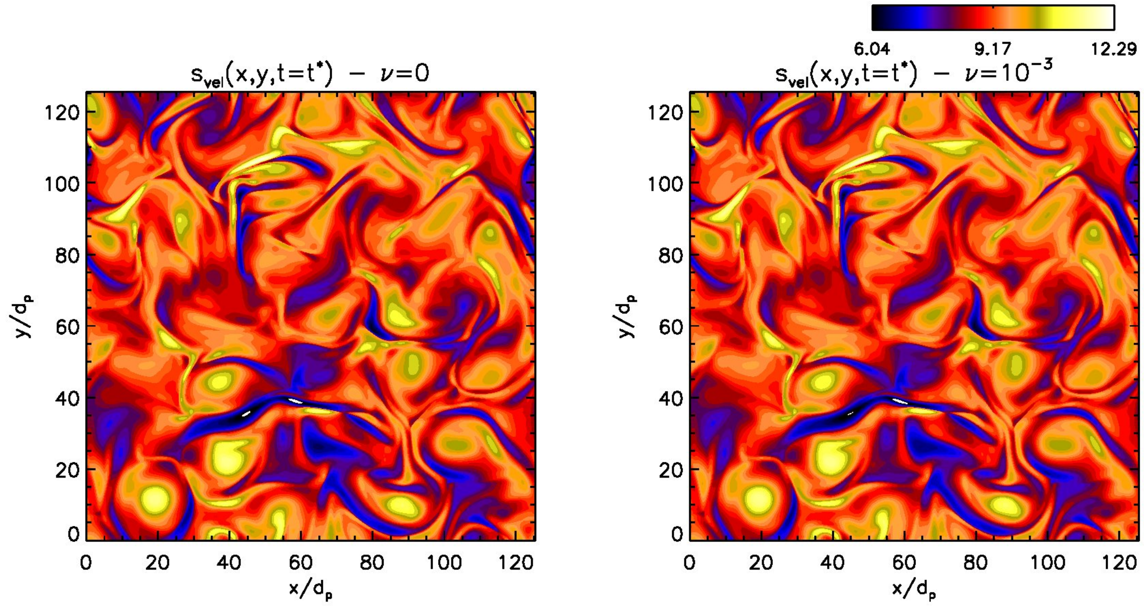


*) Not related to the thermodynamic entropy. It does not obey the H-theorem for the entropy growth.
*) It does not show differences between the two runs, i.e. it is not a good proxy for the collisional dissipation of velocity-space structures (not correlated with current sheets).

Entropy vs *velocity-space* Entropy density

$$S(t) = -\int d^3r d^3v f \log f \quad \dashrightarrow \quad s_{vel}(\boldsymbol{x}, t) = \left[n(\boldsymbol{x}) \log \left(\frac{n(\boldsymbol{x})}{\Delta v^3} \right) - \int d^3v f(\boldsymbol{x}, \boldsymbol{v}) \log f(\boldsymbol{x}, \boldsymbol{v}) \right]$$

Liang et al., 2019



*) Not related to the thermodynamic entropy. It does not obey the H-theorem for the entropy growth.

*) It does not show differences between the two runs, i.e. it is not a good proxy for the collisional dissipation of velocity-space structures (not correlated with current sheets).

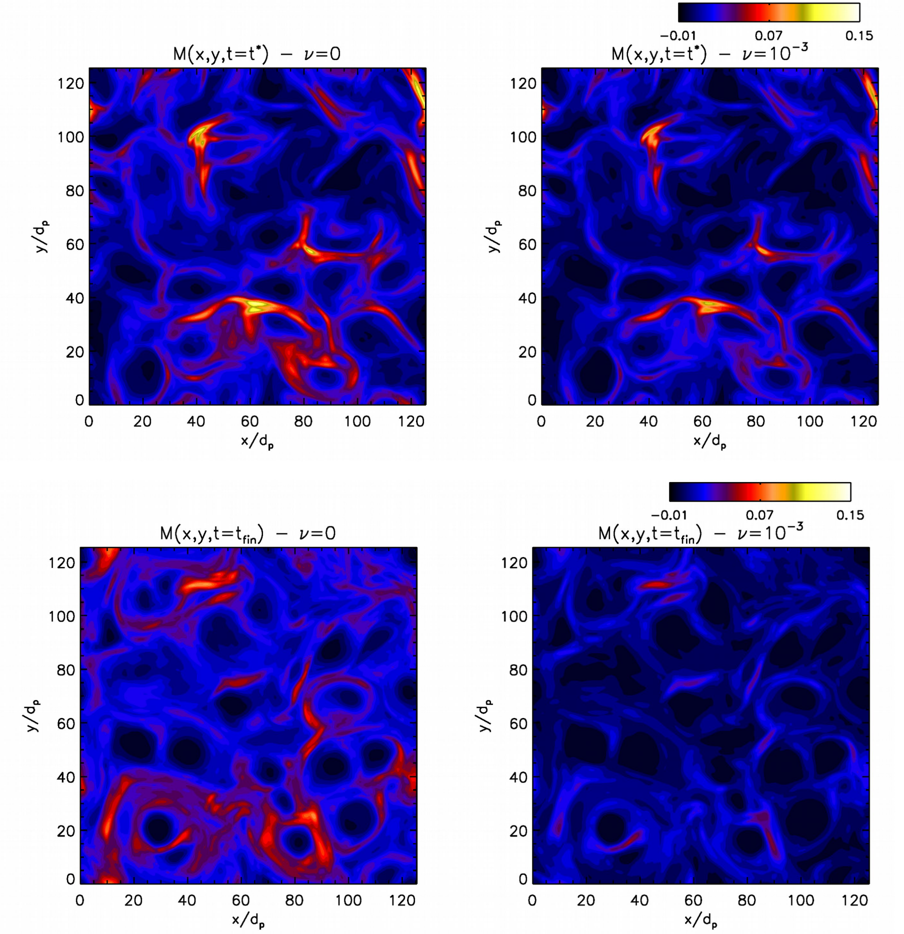
Normalized entropy density as proxy of dissipation

$$M(\boldsymbol{x}) = \frac{s_M(\boldsymbol{x}) - s(\boldsymbol{x})}{n(\boldsymbol{x})}$$

Kaufman & Peterson 2009

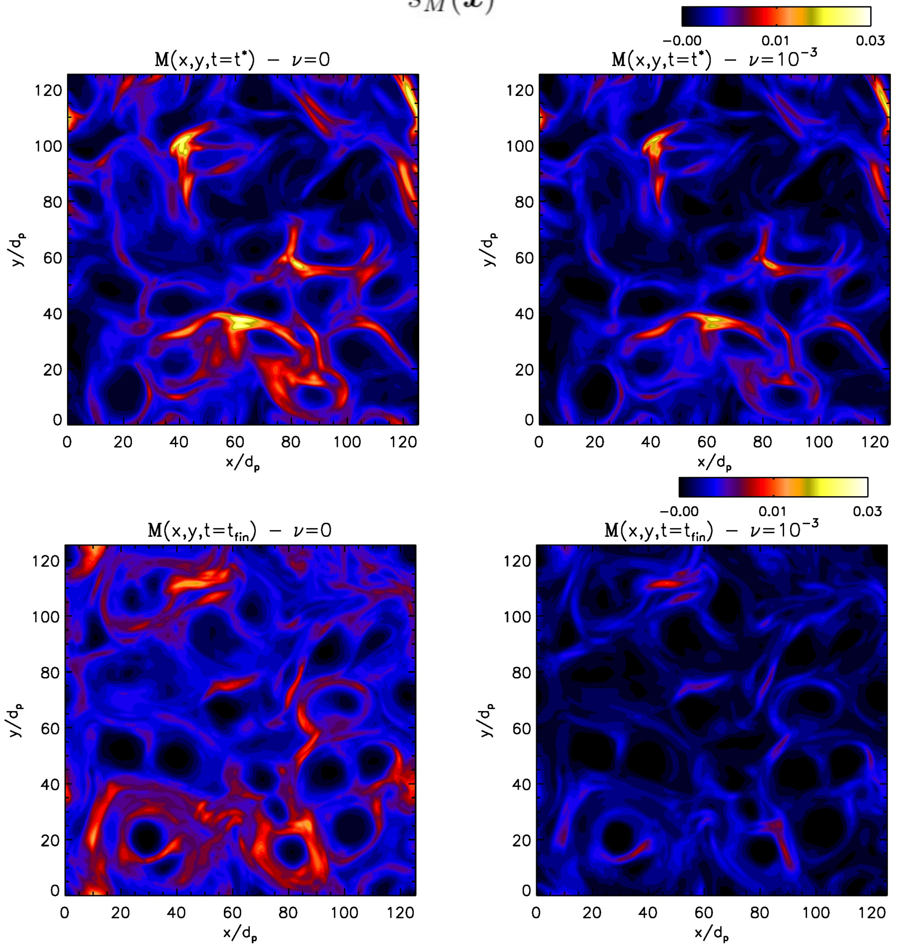
$$\mathcal{M}(\boldsymbol{x}) = \frac{s_M(\boldsymbol{x}) - s(\boldsymbol{x})}{s_M(\boldsymbol{x})}$$

Liang et al., 2019



t=t*

t=t$_{fin}$

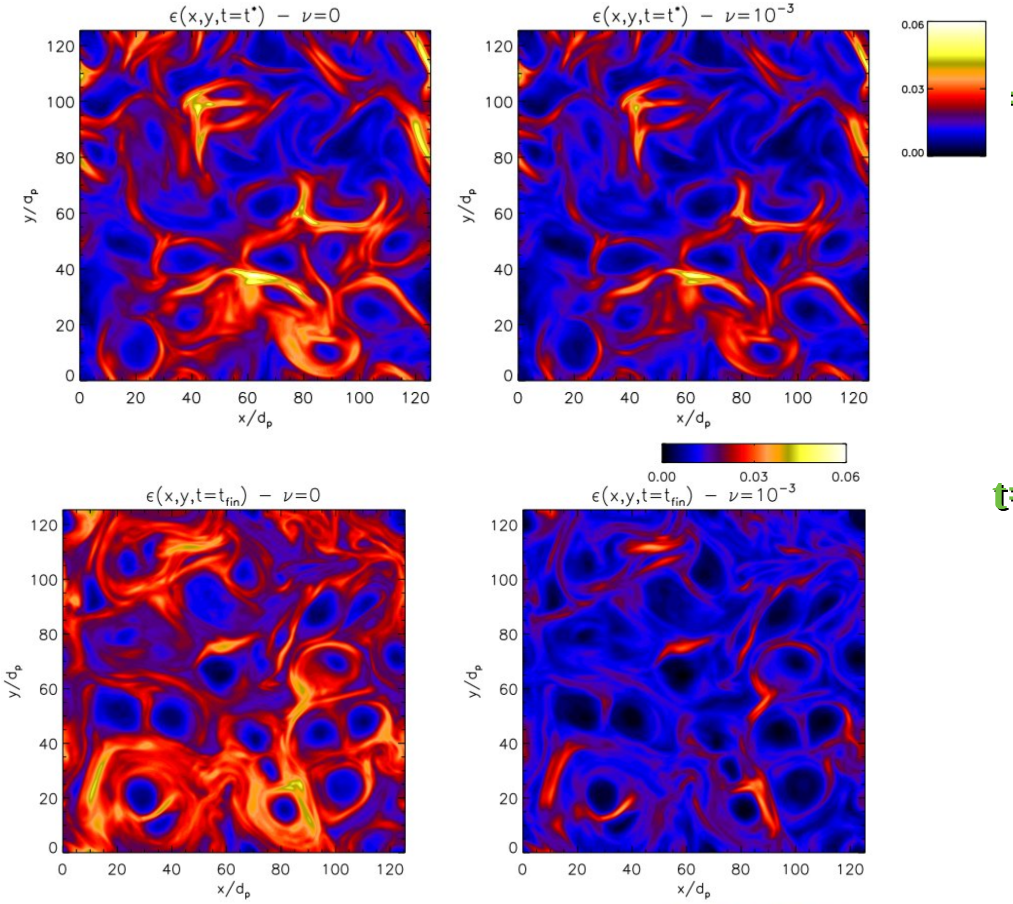


Normalized entropy density vs epsilon

$$\varepsilon(x,y,t) = \frac{1}{n}\sqrt{\int\!\!\int \left[f(\mathbf{x},\mathbf{u},t) - f_M(\mathbf{x},\mathbf{u},t)\right]^2 d^3\mathbf{u}}$$

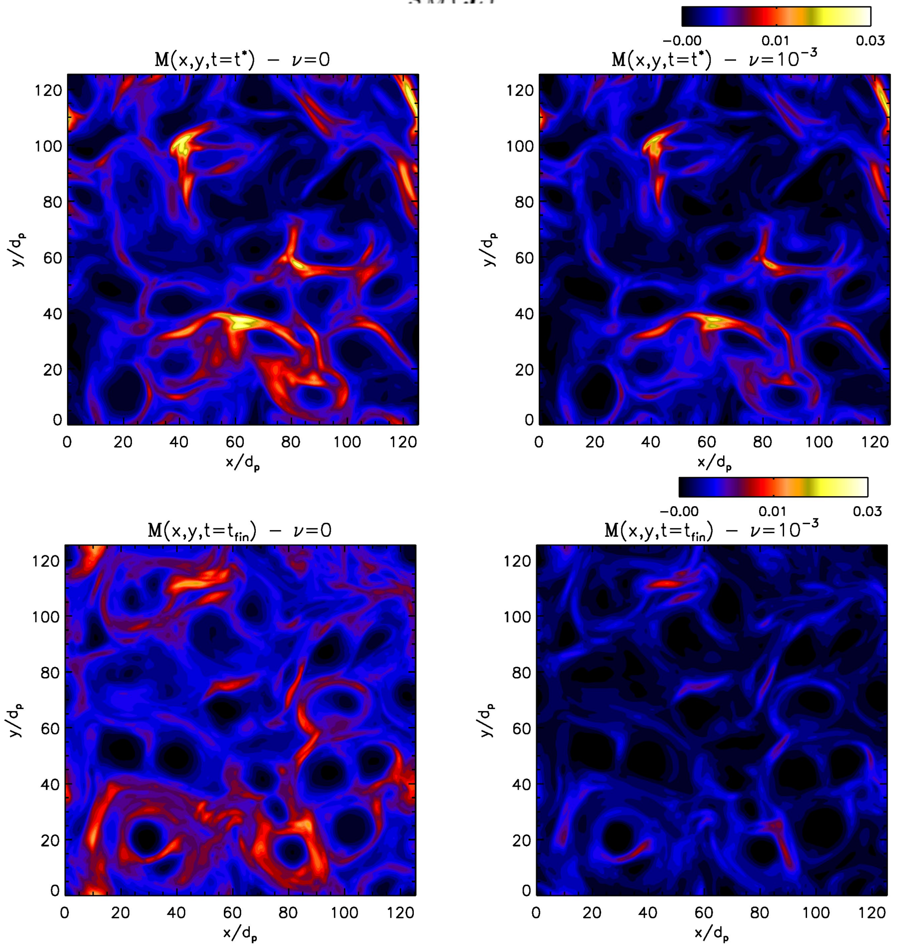
$$\mathcal{M}(\boldsymbol{x}) = \frac{s_M(\boldsymbol{x}) - s(\boldsymbol{x})}{s_M(\boldsymbol{x})}$$

Liang et al., 2019



=t*

t=t$_{fin}$



Conclusions

- **Collisions are the physical mechanism that introduces irreversibility** in the system;
- The **collisionless assumption may locally fail** also in weakly-collisional plasmas, such as the solar wind, **due to the presence of fine velocity structures**, *naturally produced by plasma turbulence at kinetic scales*, which are dissipated much faster compared to the Spitzer-Harm time.
- **Collisions could locally compete with other collisionless mechanisms in the transformation of the VDFs free energy into heat.**

$g = 10^{-10}$

$g = \nu_{pp}/\omega_{pp}$

$\tau_{NL} = 10^1\text{-}10^2\ \Omega^{-1}_{cp}$ *Matthaeus et al., APJL 2014*

$\tau_I = 1/\gamma_I = 10^2\text{-}10^3\ \Omega^{-1}_{cp}$

$\tau_{pp} = \tau_{SH} = 10^4\text{-}10^5\ \Omega^{-1}_{cp}$ (quasi-Maxwellian approach!)

$\boldsymbol{\tau_{pp} = 10^2\text{-}10^3\ \Omega^{-1}_{cp}}$ **(by retaining fine structures)**

Kinetic turbulence $\xrightarrow{\text{VDF distortions}}$ **Collisions** $\xrightarrow{\text{Dissipation of fine velocity structures}}$ **Plasma Heating**

Pezzi, *Valentini and Veltri, PRL* ***116***, *145001 (2016)*
Pezzi, *JPP* ***83***, *555830301 (2017)*

Conclusions

- First **Eulerian self-consistent hybrid Boltzmann-Maxwell** simulations suggest that:
  - Collisional effects does not affect the **statistical features of turbulence** at sub-proton scales.
  - Collisions strongly suppress the **presence of non-Maxwellian features** and the **enstrophy cascade** towards small velocity scales (~ large Hermite modes).

    … this may reduce the capability of the plasma of generating unstable fluctuations.
  - Collisions produces an the **entropy growth** but not a ***temperature increase*** . . .

    … this last aspect may change when considering also electron-proton collisions.

… Major Caveat:

Absence of a statistical theory for strongly turbulent, correlated plasmas

Collisional effects are usually introduced for plasmas close to equilibrium (~1st order correction to uncorrelated, collisionless plasmas, *Akhiezer 1975*), while turbulence is a strongly non-equilibrium dynamical state…

… this ultimately paves the way to unexplored scenarios in basic plasma physics!!

***Pezzi** et al., under review (2019)*

Extra: The choice of “proper” collisional operators

Plasmas are quite different from neutral gases!

Liouville equation

Boltzmann operator

BBGKY hierarchy

Balescu operator

Landau operator

Series expansion

- NO cut-off for high impact parameter;
- Quasi-equilibrium plasma
- Plasma Dispersion Function is taken into account

- Cut-off for high impact parameter;
- Quasi-equilibrium plasma
- Plasma Dispersion Function is NOT taken into account

- The Boltzmann operator is widely used to model plasma collisions but it is usually introduced by assuming the probability conservation (not from Liouville equation…)
- Both “natural” collisional operators for plasmas involves gradient in velocity space!
- Both “natural” collisional operators for plasmas have been derived under the assumption of quasi-equilibrium plasmas! Turbulence it's “a bit” far from equilibrium! But it is the best we can do…

Extra: The Hermite-space scaling for Landau and Dougherty operators

Landau operator

- **Non-local** effect in the Hermite space:
  … for each Hermite coefficients, all other Hermite coefficients are changed;
- Asymptotic scaling: **τ(m) ~1/m²**

Dougherty operator

- **Local** effect in the Hermite space…
  … when linearized the Lenard-Bernstein behavior is properly recovered.
- Asymptotic scaling: **τ(m) ~ 1/m**

Implications for velocity-space enstrophy cascade

I. **Enstrophy conservation.**
The enstrophy cascade is based on the global enstrophy conservation. Since a collisional operator breaks its conservation, it is difficult to determine (from fundamental perspective) the effect of collisions on the cascade itself.

II. By assuming that the enstrophy conservation is only weakly broken:
   a) The **Dougherty operator** may generate **Hermite-space spectral break** only **Vlasov equation is dominated by advection/electric terms**.
   b) The **Landau operator** may produce a spectral break in **any case**.

$$\tau_\nu(m) = \frac{1}{\sqrt{\beta}\sqrt{m}}\,\omega_{ci}^{-1}$$

$$\tau_E(m) = \frac{1}{M_t\sqrt{m}}\,\omega_{ci}^{-1}$$

$$\tau_B(m) = \frac{1}{m}\,\omega_{ci}^{-1}$$

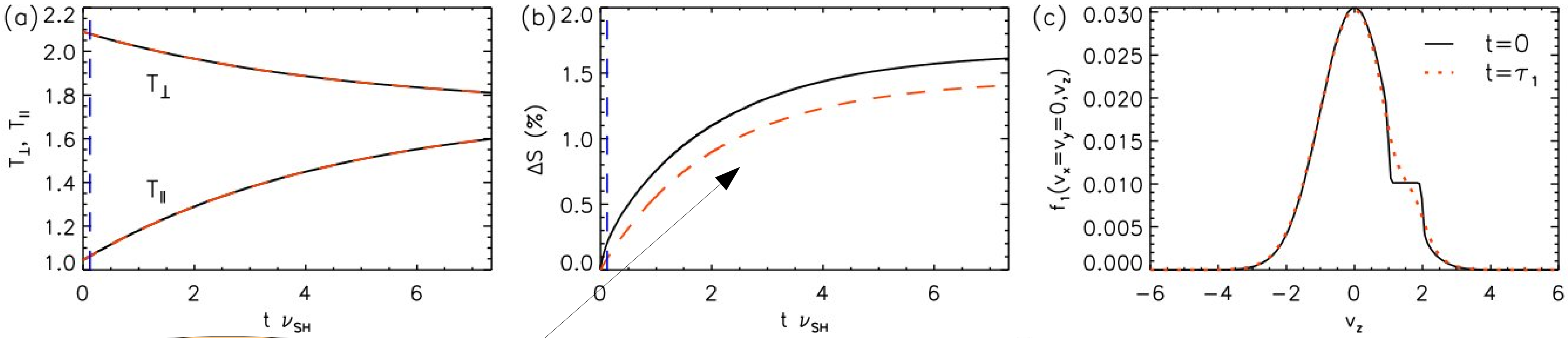
***Pezzi** et al., PPCF **61**, 054005 (2019)*

Extra: Plateau vs Anisotropies

$$f_1(\mathbf{v}) = C_1 f_{M,T_\perp}(v_x) f_{M,T_\perp}(v_y) f_{p,T_\parallel}(v_z)$$ Plateau v_z + Temp. Anisotropy

$$f_2(\mathbf{v}) = C_1 f_{M,T_\perp}(v_x) f_{M,T_\perp}(v_y) f_{M,T_\parallel}(v_z)$$ NO Plateau v_z + Temp. Anisotropy

$$N_{v_x} = N_{v_y} = 51$$
$$N_{v_z} = 1601$$
$$A = T_\perp / T_\parallel = 2$$



Multi-exponential fit

$$\Delta S(t) = \sum_{i=1}^{K} \Delta S_i (1 - e^{-t/\tau_i})$$

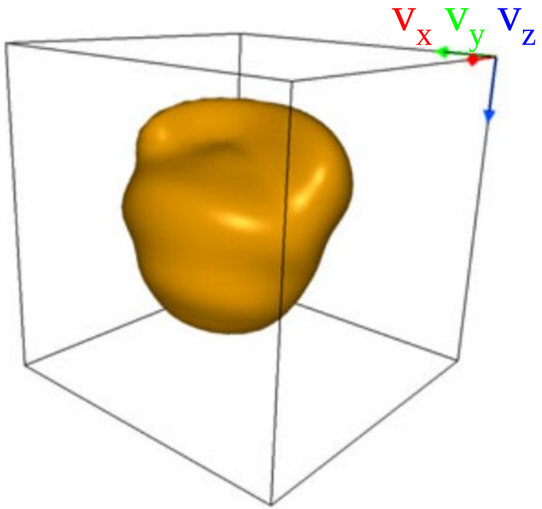
$\tau_1 = 0.14\nu_{SH}^{-1}$ → $\Delta S_1/\Delta S_{tot} = 25\%$

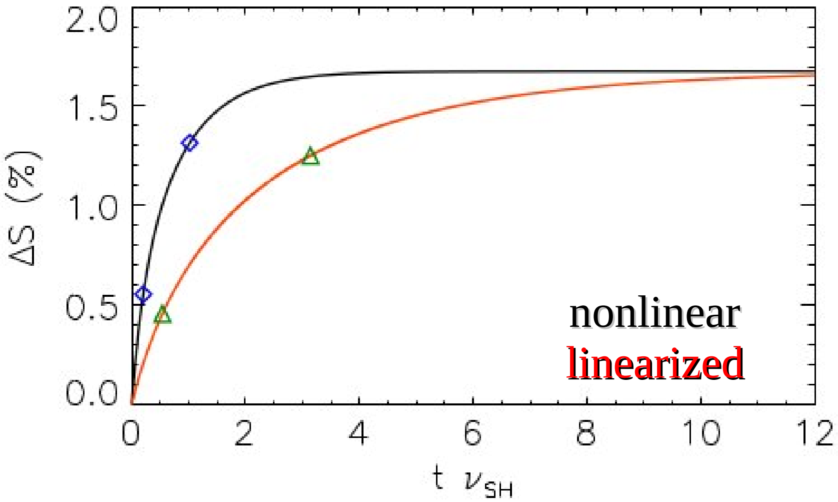
$\tau_2 = 2.03\nu_{SH}^{-1}$ → $\Delta S_2/\Delta S_{tot} = 75\%$

… the plateau is dissipated faster than global anisotropy...

Pezzi, *Valentini and Veltri, PRL* ***116***, *145001 (2016)*

Extra: Nonlinearities in the collisional operator





NONLINEAR

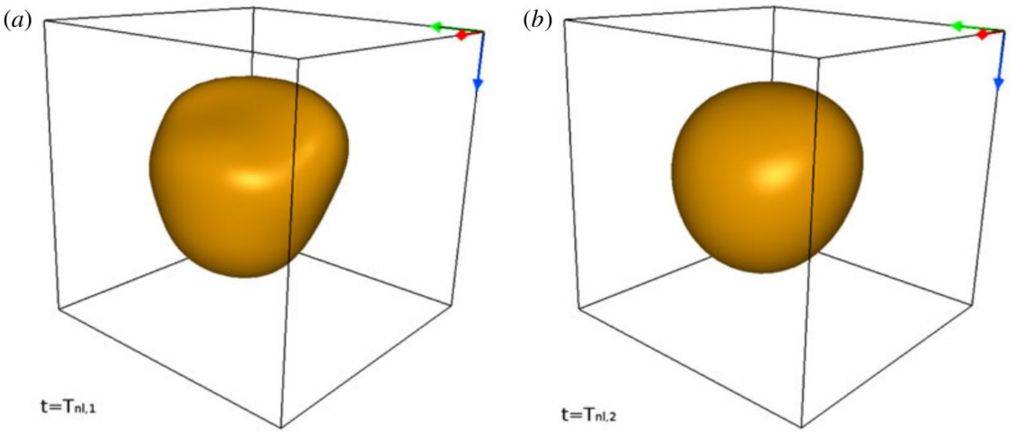
(i) $\tau_1^{nl} = 0.20\, \nu_{SH}^{-1} \rightarrow \Delta S_1^{nl}/\Delta S_{tot} = 26\,\%$

(ii) $\tau_2^{nl} = 0.82\, \nu_{SH}^{-1} \rightarrow \Delta S_2^{nl}/\Delta S_{tot} = 74\,\%$

LINEARIZED

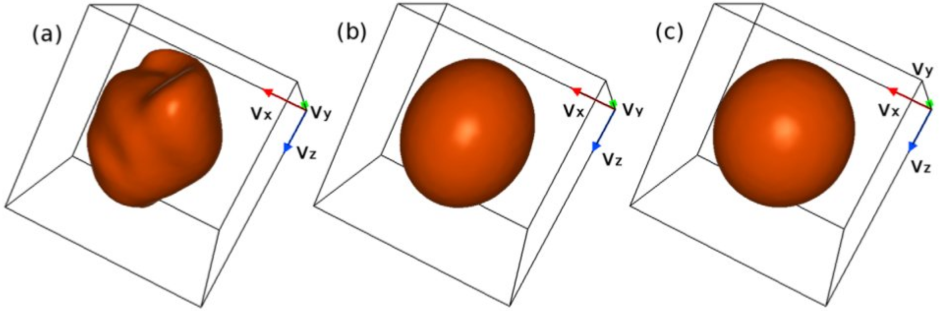
(i) $\tau_1^{lin} = 0.54\, \nu_{SH}^{-1} \rightarrow \Delta S_1^{nl}/\Delta S_{tot} = 16\,\%$

(ii) $\tau_2^{lin} = 2.60\, \nu_{SH}^{-1} \rightarrow \Delta S_2^{nl}/\Delta S_{tot} = 84\,\%$



Pezzi, JPP ***83****, 555830301 (2017)*

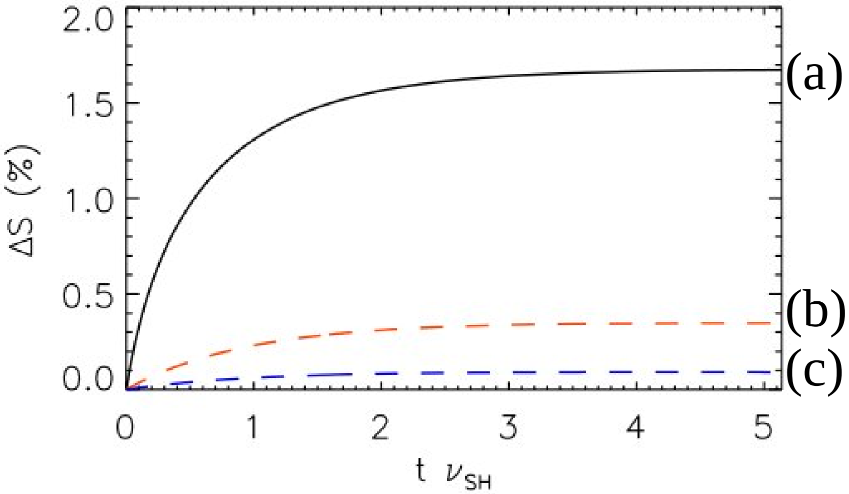
Extra: Effect of smoothing procedure on VDF



NO fit Tri-Maxwellian fit Bi-Maxwellian fit

What happens if small scales structures are artificially smoothed out by some fitting procedure? Let's focus on a turbulent, solar-wind like VDF!

Valentini et al., POP 2014



$\tau_1 = 0.20\nu_{SH}^{-1}$ → $\Delta S_1/\Delta S_{tot} = 26\%$

$\tau_2 = 0.82\nu_{SH}^{-1}$ → $\Delta S_2/\Delta S_{tot} = 74\%$

Smaller characteristic times are not recovered if small scales are artificially smoothed out

…high-resolution phase space measurements of the VDF are crucial to properly identify and take into account dissipative processes …

Pezzi, *Valentini and Veltri, PRL* ***116****, 145001 (2016)*