Collective Plasma Structures with kinetic nonlinearity: their stability criteria and interactions

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24th July 2019

Collective plasma structure formation- Electron Hole.

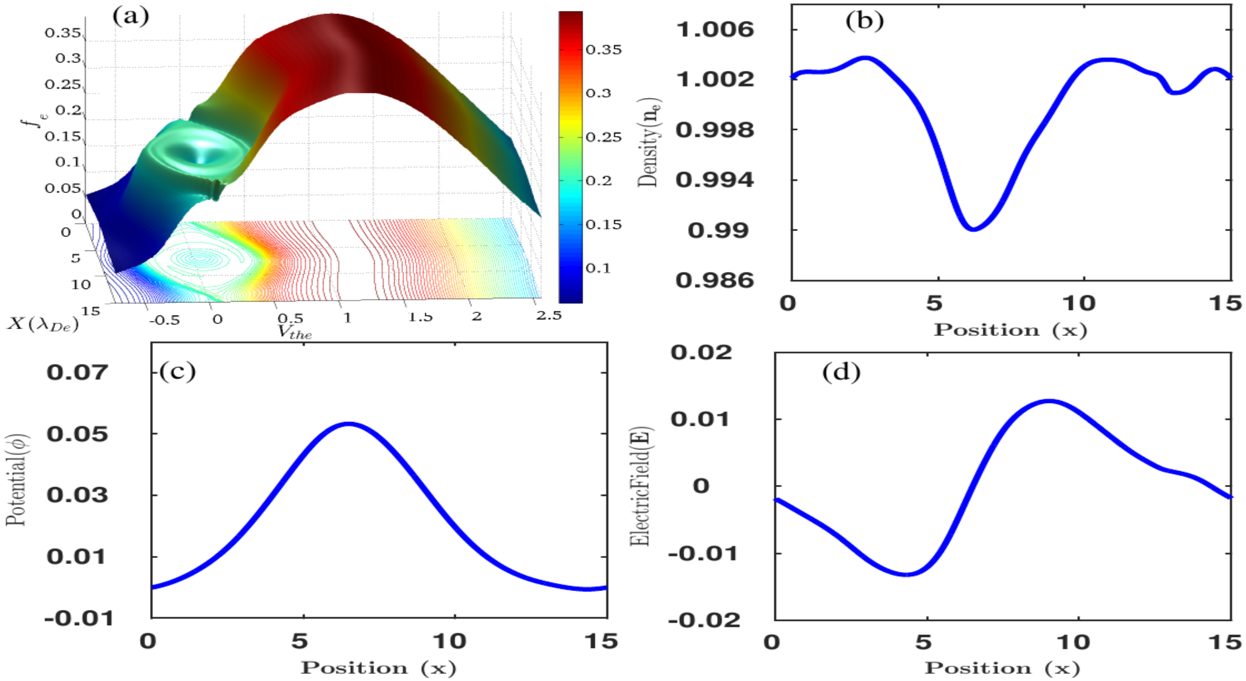
- **Coherency:**

Balance between dispersion and nonlinearity generates many coherent structures.

- In collision-less plasmas trapped particle nonlinearity generates different coherent structures: Solitary electron hole, Conoidal electron hole, double layer, etc.

Electron Hole:

A trapped particle nonlinearity generated coherent structure with reduce phase-space density in trapped electron orbit and having electron density lower than the surrounding.



Motivations and outline

- Electron holes are observed in many basic plasma experiments in many space plasma observations and fusion plasmas.

- An important agent in turbulent plasma.

Outlines

1. **the mechanism of electron hole formation from a very small amplitude seed like fluctuation in subcritical regime,**

2. **their stability criteria: forbidden region of electron hole**

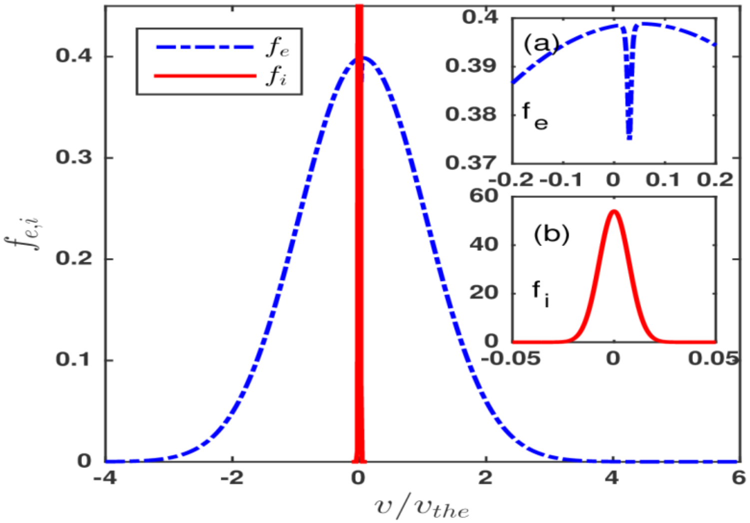
3. **interactions between two electron holes**

Generation of electron hole in subcritical regime of plasmas

Initial distribution function: A tiny Eddy like seed fluctuation electron perturbation

$$f_e(x,v) = (1 + f_1(x,v))\exp\left[-\frac{m_e(v - u_e)^2}{2T_e}\right]$$

$$f_1(x,v) = -\epsilon\,\mathrm{sech}\left[\frac{v - v_1}{L_1}\right]\mathrm{sech}^4(kx)$$



$v \to v_{the}$, $x \to \lambda_{De}$, $m \to m_e$

Simulation: v_1=0.01 & 0.05, L_1=0.003, ε= 0.02, k=0.1, T_e/T_i = 10 and u_e=0.05 which is well inside the linearly stable regime or subcritical regime of ion acoustic instability.

Vlasov-Poisson Eq. in 1D

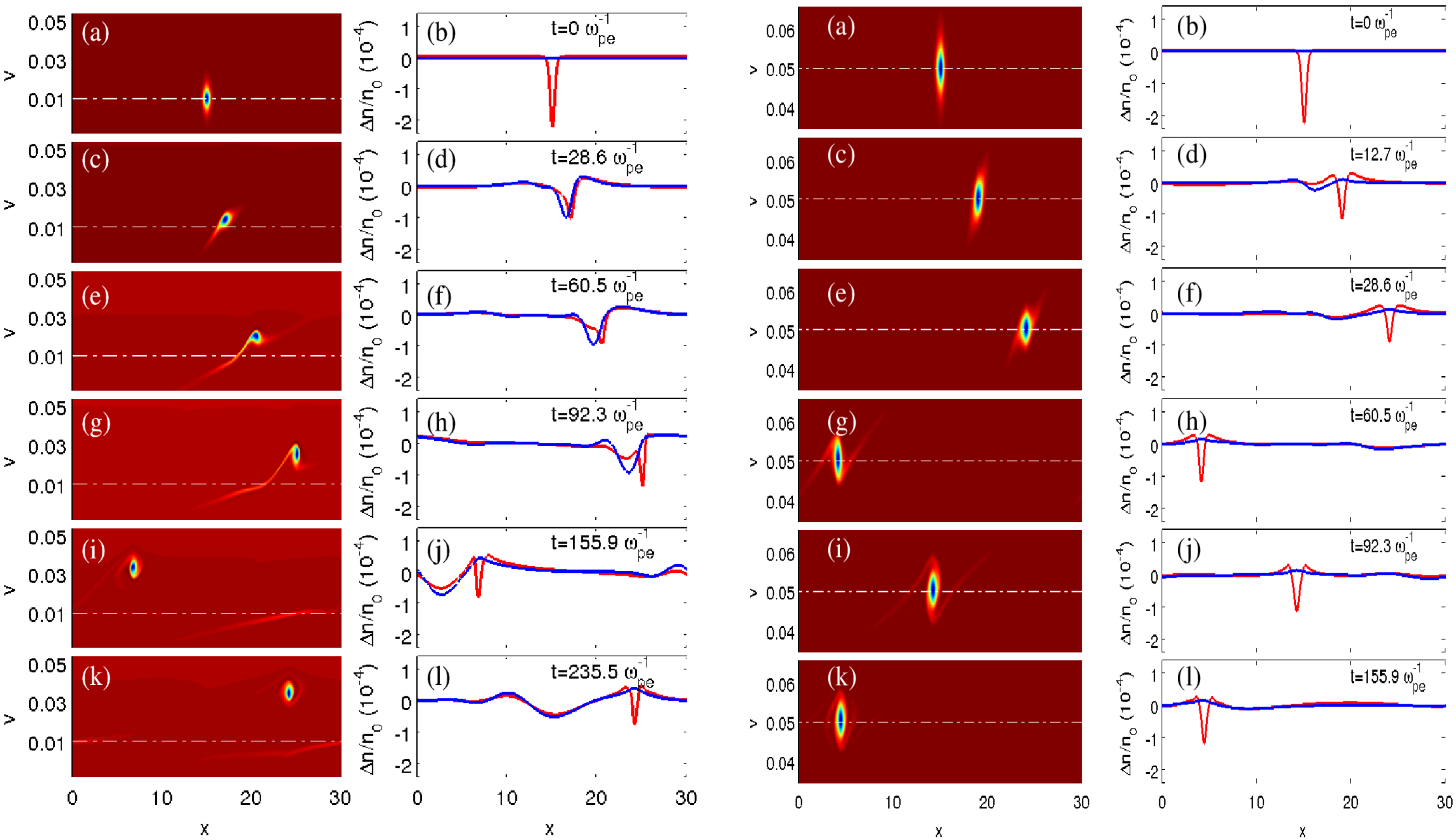
$$\left(\frac{\partial}{\partial t} + v_x\frac{\partial}{\partial x} + \frac{q_\alpha}{m_\alpha}E_x\frac{\partial}{\partial v}\right)f_\alpha = 0, \quad \text{and} \quad \frac{\partial^2\phi}{\partial x^2} = \frac{e}{\epsilon}(n_e - n_i)$$

Solve the Vlasov-Equation using Flux-Balance technique

1. Formation of electron hole

Electron hole is accelerated with initial perturbation location at $v_1 = 0.01\ v_{the}$ but not accelerated in case of $v_1 = 0.05\ v_{the}$.

$\boldsymbol{v_1 = 0.01\ v_{the} < C_s}$ $\boldsymbol{v_1 = 0.05\ v_{the} > C_s}$



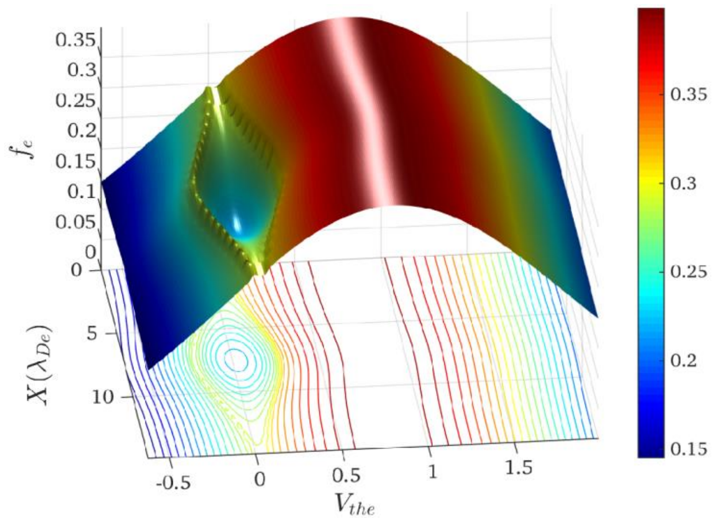
[D. Mandal, D. Sharma and H. Schamel, New Journal of Physics, 20, 073004, 2018]

Schamel's Model of electron hole & NDR

Schamel's Nonlinear model

$$f_e(x,v) = \frac{1+K}{\sqrt{2\pi}} \begin{cases} \exp\left[-\frac{1}{2}\left(\sigma_e\sqrt{2\epsilon_e} - \tilde{v}_D\right)^2\right], & \epsilon_e > 0, \text{ Free Electron} \\ \exp\left(-\tilde{v}_D^2/2\right)\exp\left(-\beta\epsilon_e\right), & \epsilon_e \le 0, \text{ Trapped Electron;} \end{cases}$$

$\epsilon_e = \frac{v^2}{2} - \phi$, σ is the step function



Substituting $f_e(x,v)$ in Poisson equation $\rightarrow NDR$

- **Nonlinear Dispersion Relation (NDR) for current driven system (No ion trapping)**

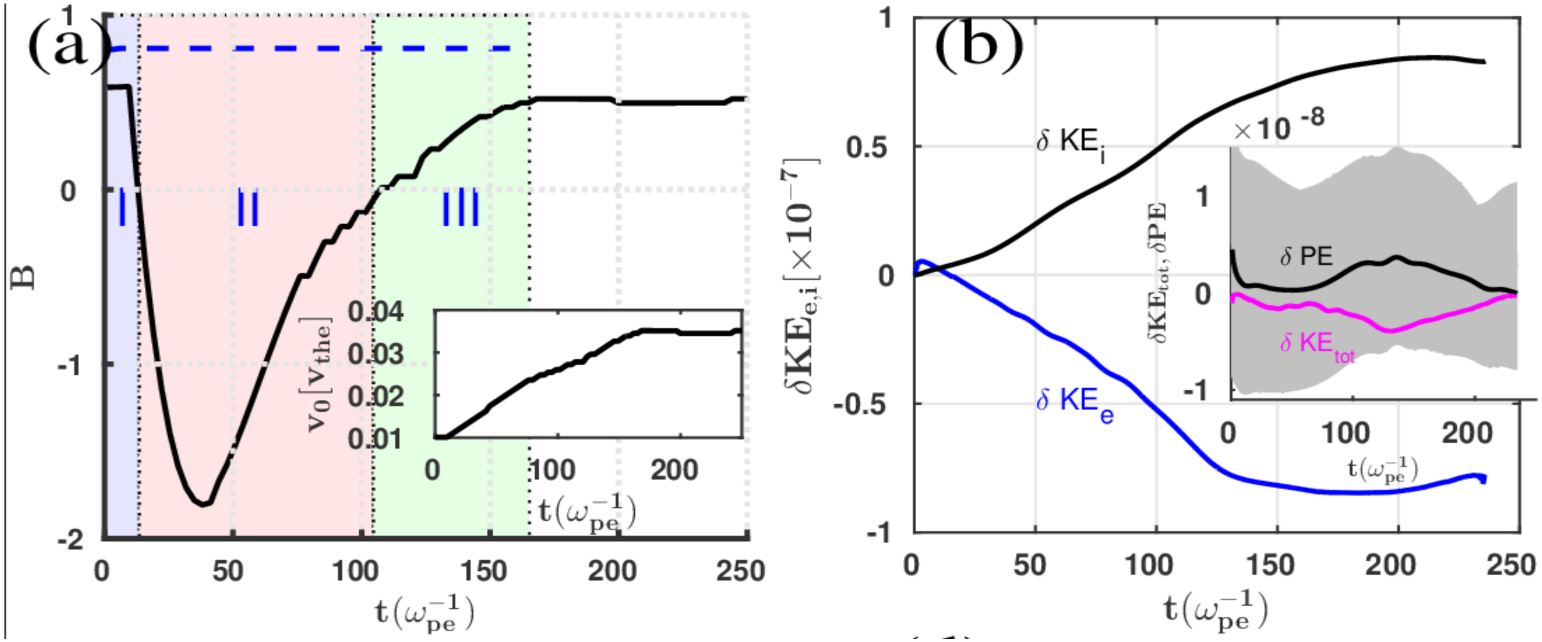
$k_0^2 - \frac{1}{2}Z_r'\left(\frac{v_0}{\sqrt{2}}\right) - \frac{1}{2}Z_r'\left(\frac{u_0}{\sqrt{2}}\right) = B$; where, $u_0 = \sqrt{\frac{m_i}{m_e} \times \frac{T_e}{T_i}}\, v_0$;

. $B = \frac{16\sqrt{\psi}}{15\sqrt{\pi}}(1 - \beta - v_0^2)\exp\left(\frac{v_0^2}{2}\right).,\rightarrow$ **Trapped particle contribution** For valid electron hole sol. **B >0**

Free particle contribution: $Z_r\left(\frac{v_0}{\sqrt{2}}\right)$ is the plasma dispersion function

- **Energy of the electron hole (ΔW < 0)**

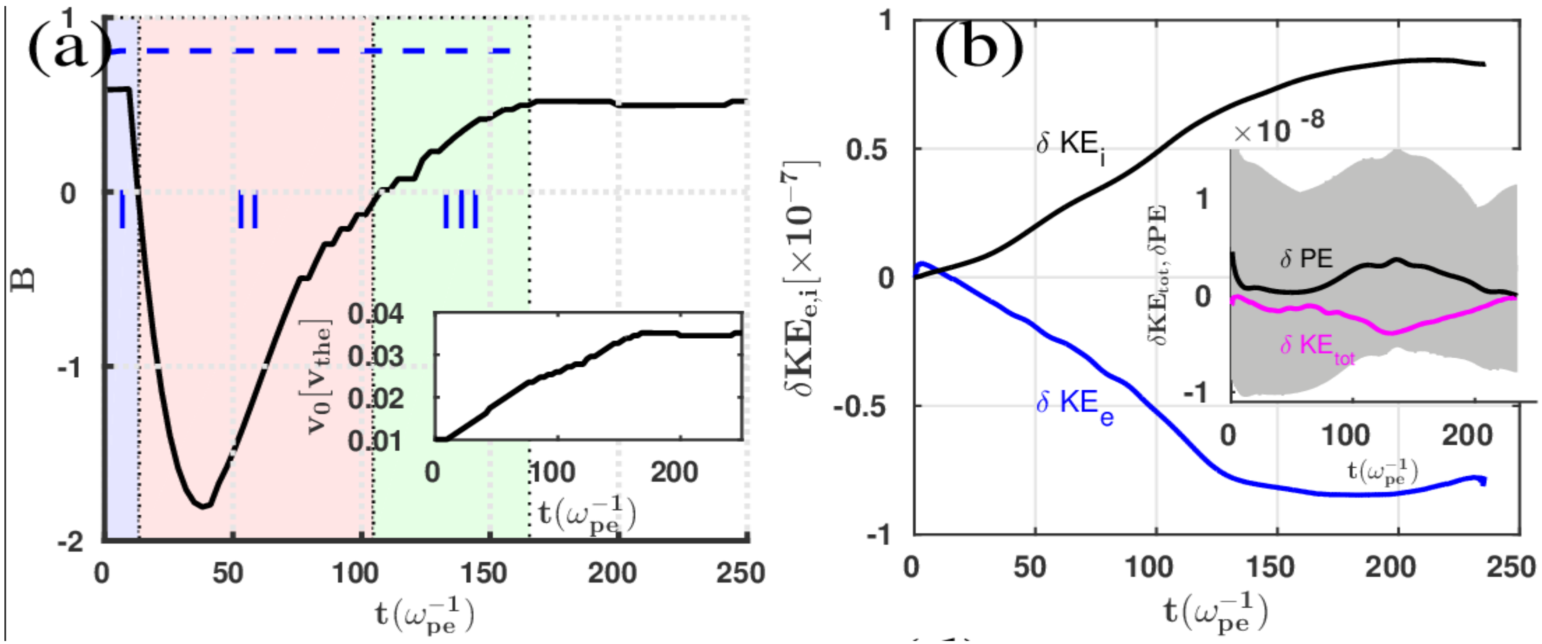
$\Delta W = \frac{\psi}{2}\left[1 + \frac{1}{2}Z_r'\left(\frac{u_0}{\sqrt{2}}\right)(1 - u_0^2)\right]$; Change of energy with respect to the unperturbed sate.



$$k_0^2 - \frac{1}{2} Z_r' \left(\frac{v_0}{\sqrt{2}}\right) - \frac{1}{2} Z_r' \left(\frac{u_0}{\sqrt{2}}\right) = B;$$

- **Three regions in Nonlinear Disp. Rel. (NDR).**
  -I and -III, B>0. But, ΔW is positive in region- I, therefore -I is unstable.
  -II have B<0 , therefore unstable.
- Both -I and -II are the forbidden regions for the Electron Hole.
- During this unstable evolution electrons lose it's energy and ions gain energy

Acceleration Mechanism: Unshielded electric field



- In region III : electric field associated with the electron hole is perfectly Debye shielded → **Stable**

- In region II: the electric field remain unshielded → **Unstable**
  → EH accelerates to higher velocity to make it stable.

3. Interaction of two solitary electron holes

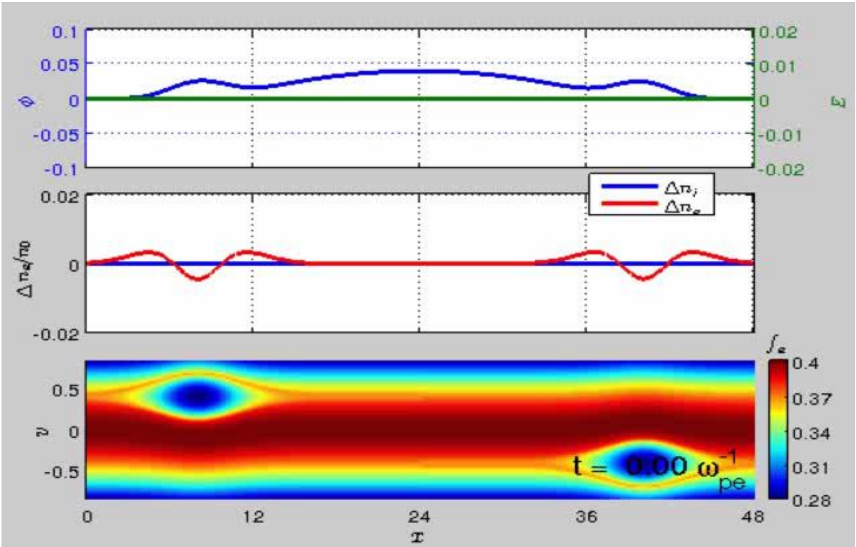
$$f_e(x,v) = \frac{1+K}{\sqrt{2\pi}} \begin{cases} \exp\left[-\frac{1}{2}\left(\sigma_e\sqrt{2\epsilon_e} - \tilde{v}_D\right)^2\right], & \epsilon_e > 0, \text{ Free Electron} \\ \exp\left(-\tilde{v}_D^2/2\right)\exp\left(-\beta\epsilon_e\right), & \epsilon_e \leq 0, \text{ Trapped Electron;} \end{cases}$$

Case	Ψ_1, Ψ_2	v_{01}, v_{02}	Δv_0	$S = \frac{\sqrt{\psi_1} + \sqrt{\psi_2}}{\Delta v_0}$
I	0.04, 0.04	0.4, -0.4	0.8	0.5
II	0.04, 0.02	0.4, -0.4	0.8	0.42
III	0.04, 0.02	0.6, 0.3	0.3	1.1
IV	0.04, 0.02	0.4, 0.3	0.1	3.4

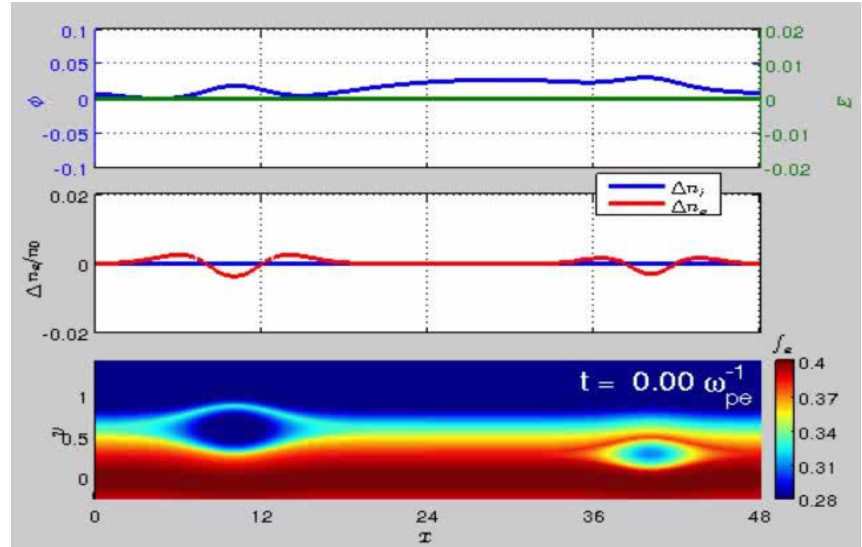
Ψ_1 and Ψ_2 $\rightarrow$ maximum potential amplitude of electron holes.
v_{01} and v_{02} $\rightarrow$ initial phase velocity of EH.
$\Delta v_0 \rightarrow v_{01}$-$v_{02}$
S $\rightarrow$ Overlap parameter

Ch: 4- Interaction of two solitary electron holes on EAW branch

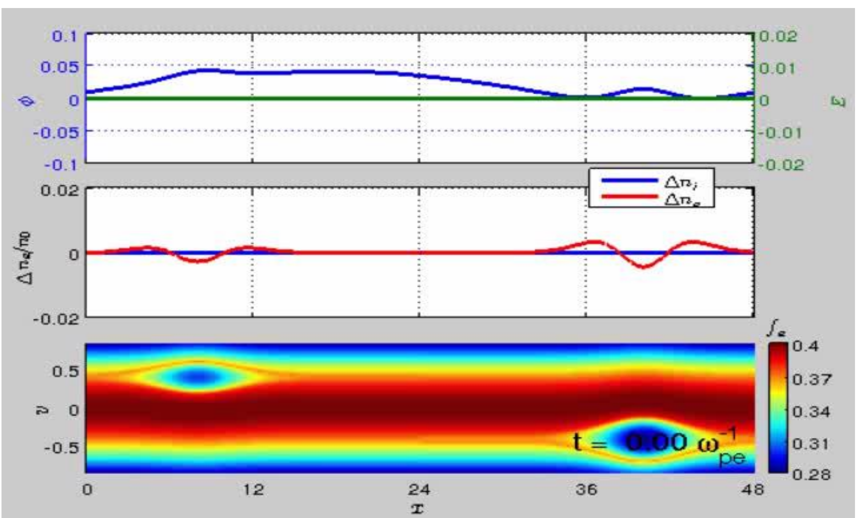
Case-I: Counter propagating SEH (S <1)



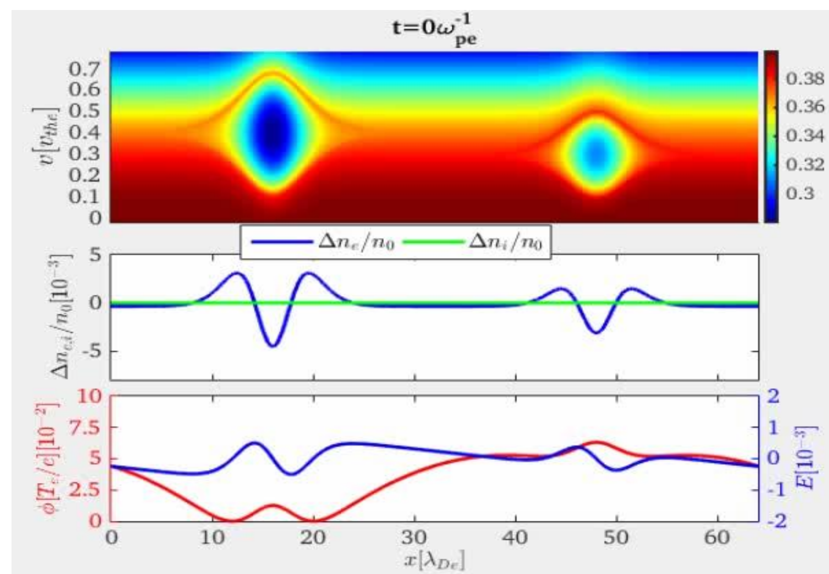
Case-III: Co-propagating SEH (S ~1)



Case-II: Counter propagating SEH (S <1)



Case-IV: Co-propagating SEH (S ~ 3)

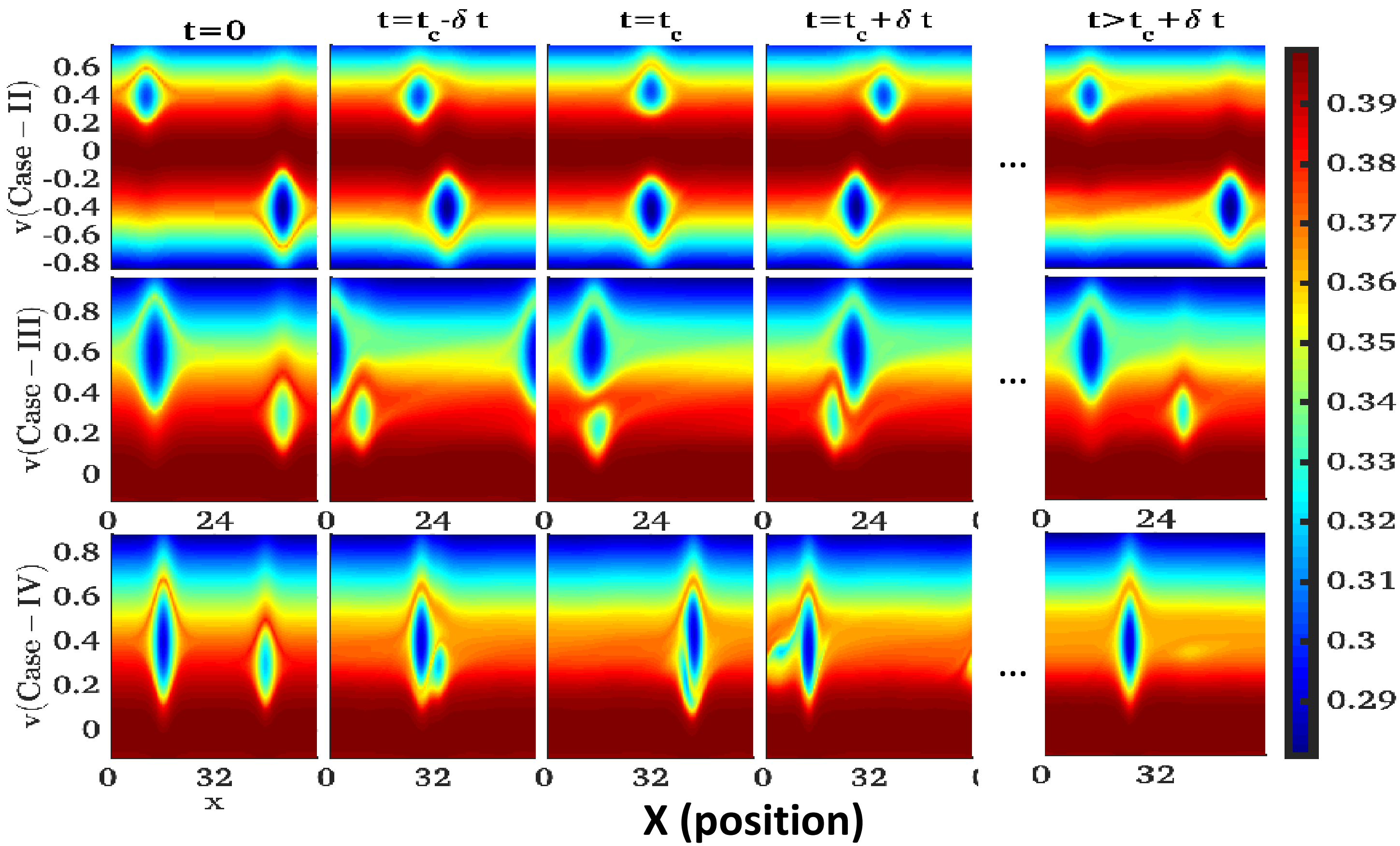


[D. Mandal and D. Sharma, Phys. Of Plasmas, **23**, 22108 (2016)]

- **Adiabatic Invariant:**
- Case I-III : Dynamics of the trapped particles remains adiabatically invariant (bounce frequency, ω_{be}^{-1}> time of interaction between two holes) (S≤ 1)→ preserves their identity

- Case IV: not preserved (S =3)→ Separatrix and the trapped particle orbits one structure are destroyed.

→ They come closer to each other after every interaction → slowly merge to form a single hole.



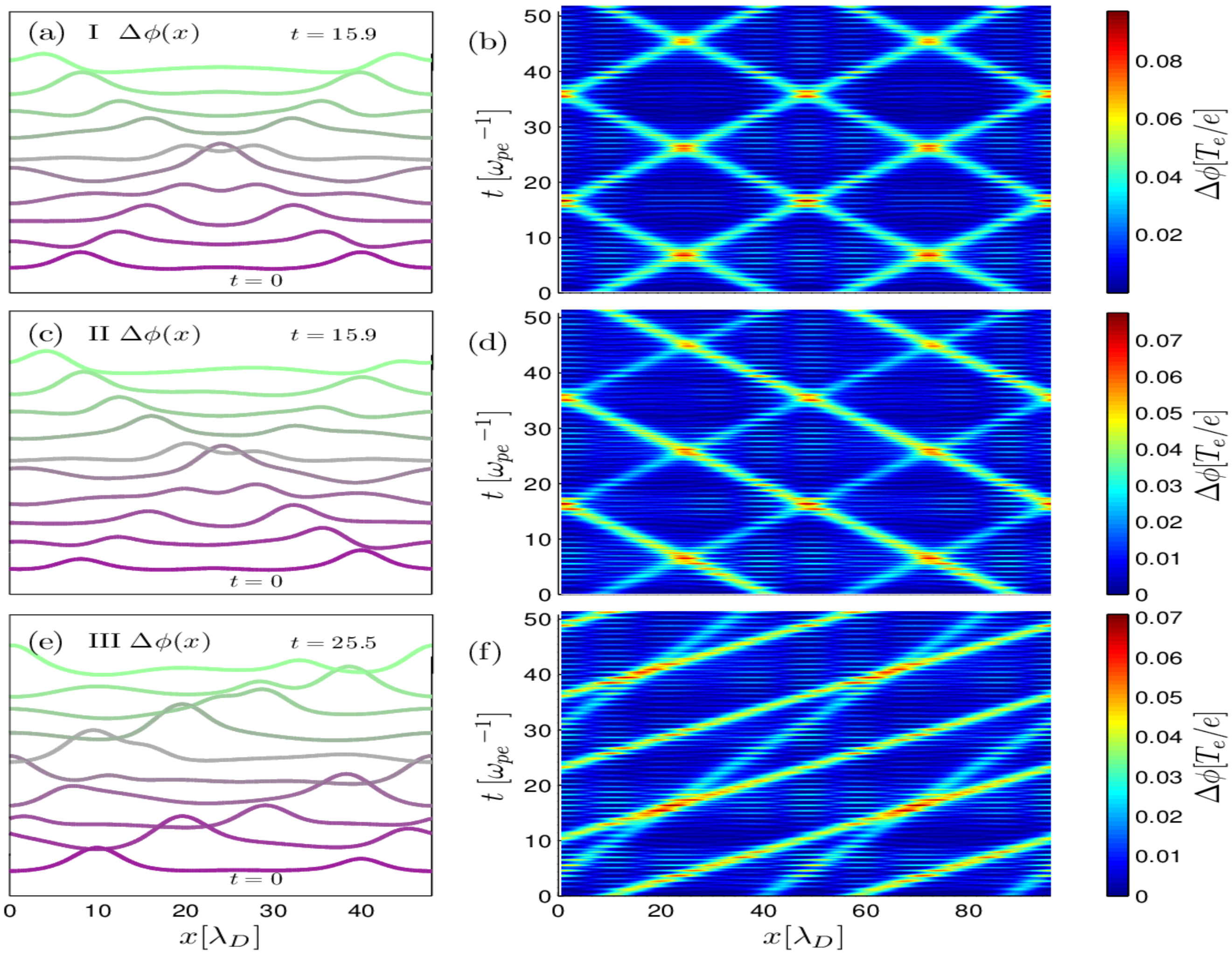
Potential time evolution: Phase shift

- In simulation we found linear plasma modes + nonlinear electron hole.
- After extracting the potential associated with linear modes we get the

S<1

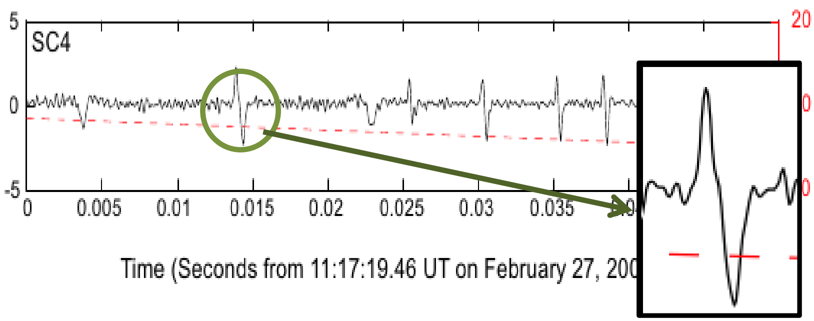
S<1

S~1

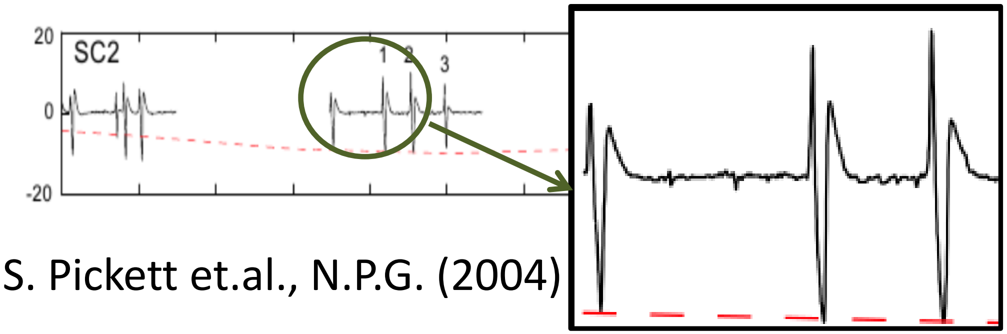


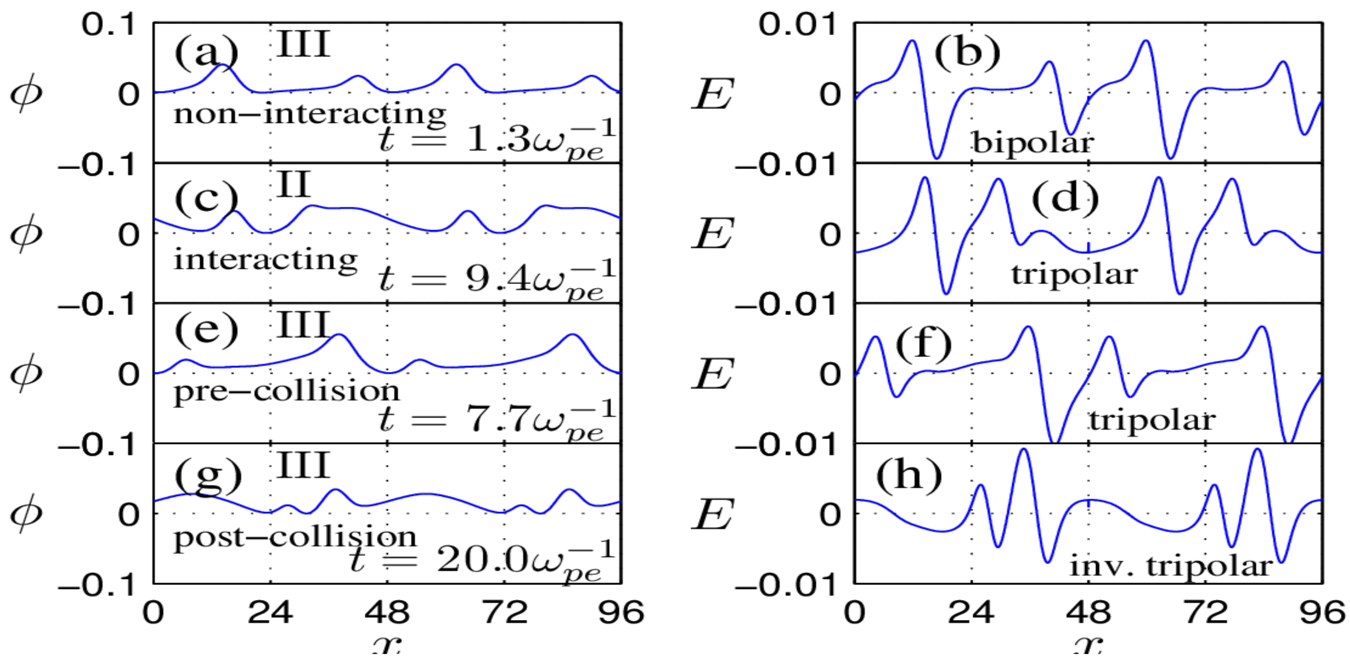
Space plasma observation

Space Plasma Observation in Auroral zone. Bipolar and Tripolar electric field pulse.



J.S. Pickett et.al., N.P.G. (2004)





frequent

infrequent

infrequent

infrequent

- The bipolar pulse corresponds to solitary electron hole and tripolar pulses are generated during the time of interaction of two SEHs.

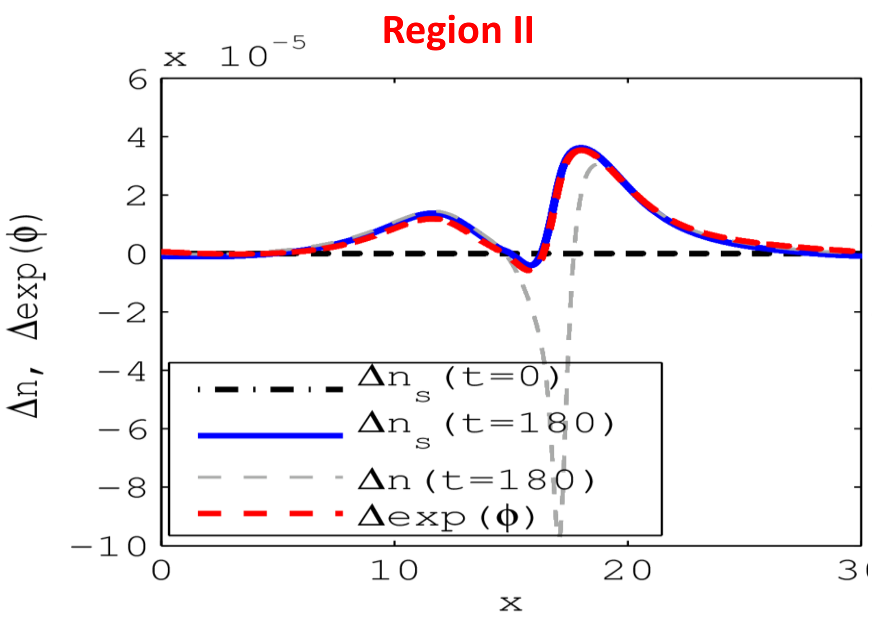
[D. Mandal and D. Sharma, Phys. Of Plasmas, **23**, 22108 (2016)]

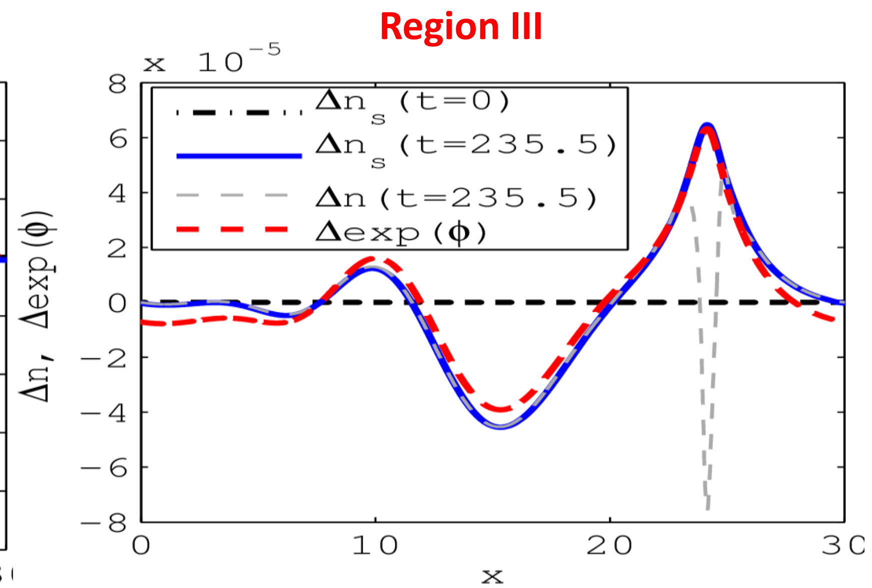
Conclusions

- Electron holes are generated from a small eddy like seed fluctuation in linearly subcritical regime of plasmas.
- Electron holes follow a nonlinear dispersion relation (NDR): nonlinear eigen modes
- We identified an unstable forbidden region of these nonlinear eigen modes .
- In the unstable region the electron hole mediates the energy exchange mechanism between electron thermal energy and ion acoustic non-uniformity.
- For the cases, the overlap parameter $S < 1$, the dynamics of trap electrons remains adiabatically invariant during the time of interaction: Structures interact like solitons.
- For the case $S \gg 1$, their dynamics does not remain adiabatically invariant and structures merge into a single structure.
- Finite phase shift is detected during their crossing.
- Bipolar and tripolar electric pulses are generated during their interactions.


Thank you

Acceleration Mechanism: Non-uniform electron Pressure





- In region III (Stable Electron Hole), the streaming electron pressure (exp^{φ}) is symmetric around the hole and associated electric field is perfectly Debye shielded.
- Electron pressure is asymmetric for an unstable hole (region II) ; there is a net gain of ion flux into the electron hole.
- So, n_e-n_i is increased at the hole location, which increases potential => Increases velocity.

Vlasov Equation and Subcritical regime of plasmas

Kinetic Description of plasma is governed by Vlasov equation and Poisson Equation

Vlasov-Poisson Eq. in 1D

$$\left(\frac{\partial}{\partial t} + v_x \frac{\partial}{\partial x} + \frac{q_\alpha}{m_\alpha} E_x \frac{\partial}{\partial v}\right) f_\alpha = 0, \quad \text{and} \quad \frac{\partial^2 \phi}{\partial x^2} = \frac{e}{\epsilon}(n_e - n_i)$$

Subcritical regime of Plasmas:

According to linear Vlasov theory of plasma below a critical drift value between electron and ion, linear plasma modes never be unstable.

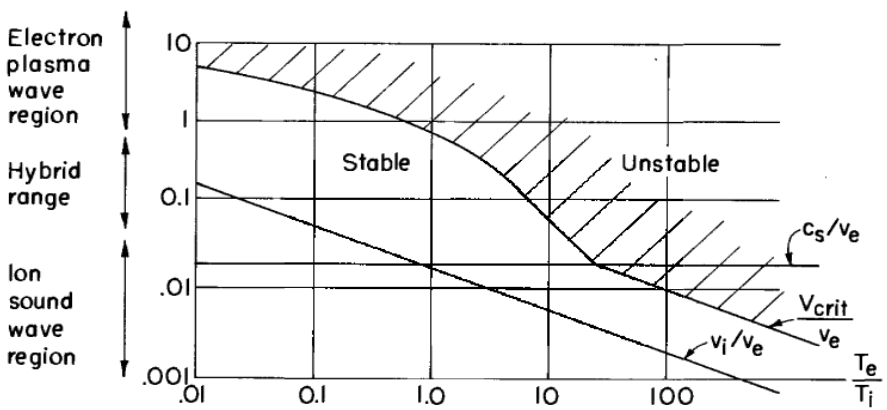


FIGURE 9.7.1
Plot of critical drift velocity as a function of T_e/T_i for the onset of electrostatic instability in an electron-ion plasma. (*After B. D. Fried and R. W. Gould, Phys. Fluids*, **4**:139 (1961).]

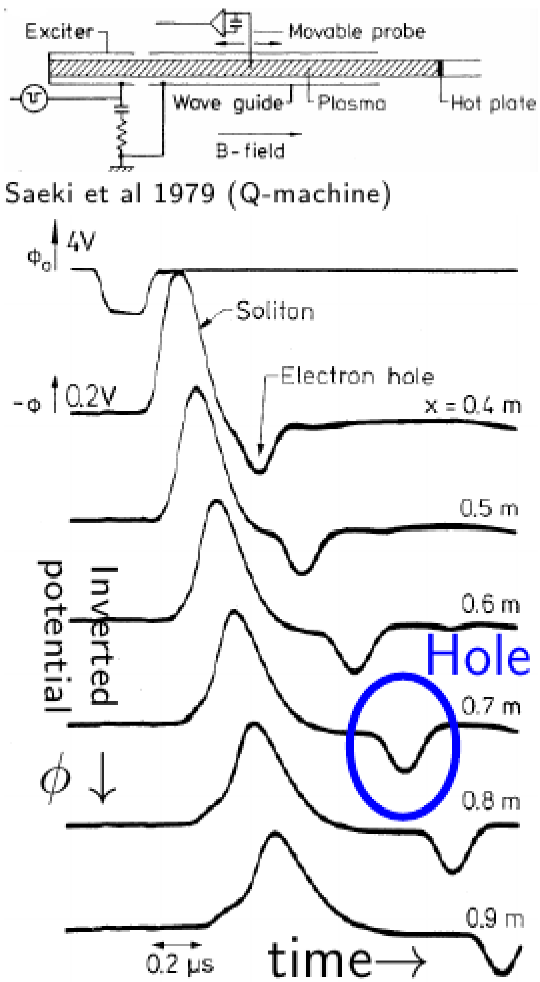
Linear Stability Criterion:

For $T_e/T_i = 10$

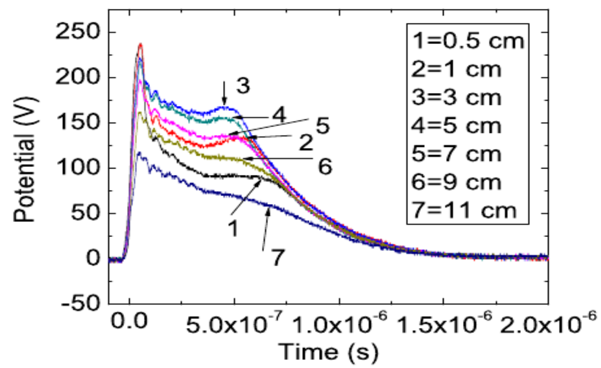
Drift velocity $v_D < 0.056\ v_{the}$.

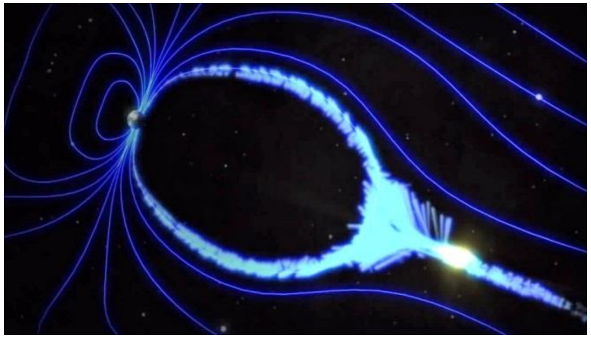
Observation of kinetic collective structures: Electron holes

Experimental and space plasma observation

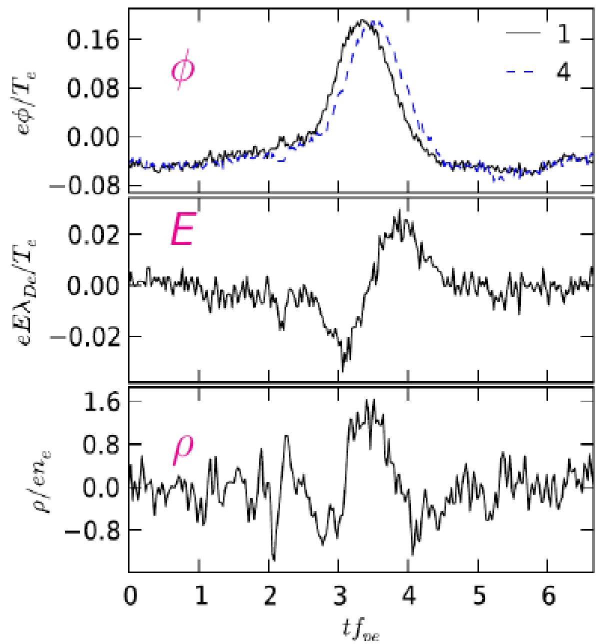


S. Kar *et. al.* At IPR





Lefebvre et al 2010 (LAPD)



Matsumoto et al 1994