Exact hybrid-Vlasov equilibria for sheared plasmas with uniform magnetic field

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Outline

1. Introduction.

2. Exact stationary solutions for a shear flow:

    a) B *parallel to* u;

    b) B *perpendicular to* u;

    c) *oblique B.*

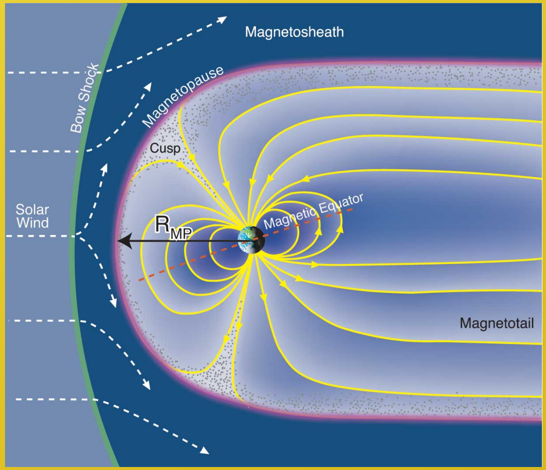
3. Numerical simulations.

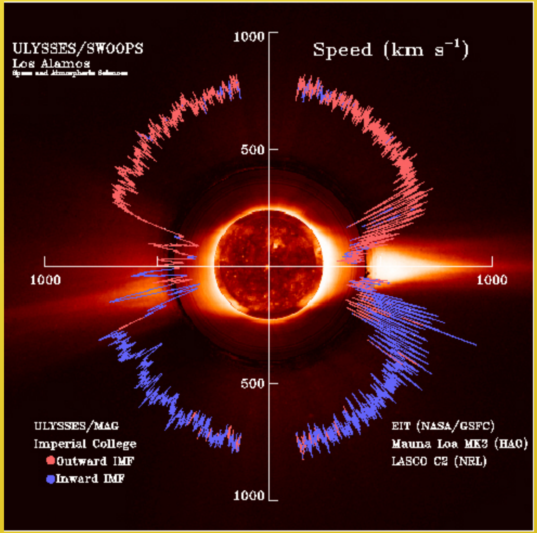
4. Conclusions.

1. Introduction

Magnetized plasmas with a <u>sheared flow</u>
are present in several natural contexts:

- **Solar wind - planetary magnetosphere interface**





- **Fast-slow solar wind interface**

- **Astrophysical jets**

Stationary shear flows in collisionless plasmas

a non-trivial problem in kinetic theory

e.g, a “shifted Maxwellian”

$$f(x,\mathbf{v}) = Ce^{-\frac{m_p}{2\kappa_B T}\left\{v_x^2+[v_y-v_0(x)]^2+v_z^2\right\}}$$

is **<u>not</u>** a stationary distribution function

- **Fully kinetic** exact solutions:
  - <u>parallel</u> **B** (Roytershtein & Daughton, PP, 2008);
  - <u>perpendicular uniform</u> **B** (Ganguli et al., PhF, 1988; Nishikawa et al., PhF, 1988; Cai et al., PhFB, 1990);
  - <u>perpendicular, non uniform</u> **B** (Mahajan & Hazeltine, PhPl, 2000).

At scales $l \approx d_i$: the **<u>Hybrid Vlasov-Maxwell (HVM)</u> model**

(e.g., Valentini et al., JCPh, 2007; Franci et al., ApJL, 2015;...):

a) ***kinetic description for ions*** (Vlasov equation);
b) electrons treated as ***a massless fluid***.

- ***Within the HVM approach:***
  “extended fluid model” (Cerri et al., PhPl, 2013): an <u>approximately stationary shear flow</u>.

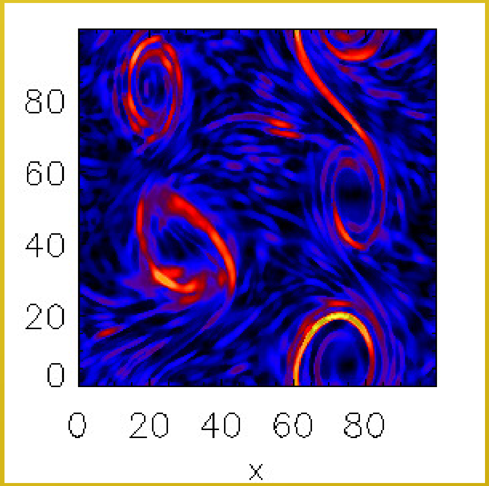
Our approach

Finding <u>stationary</u> solutions of the **HVM equations**, representing a <u>magnetized shear flow</u>, in various magnetic configurations (Malara et al., ApJ 2018).

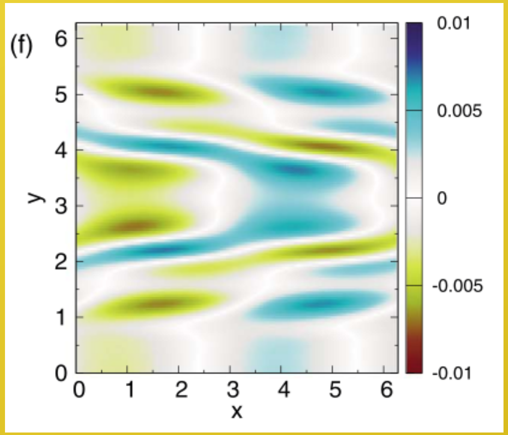
- valid for shear width $\Delta x \approx d_I, R_L$;
- <u>exact</u>, within HVM approach.

Applications:

- <u>Kelvin-Helmohotz instability</u> (see A. Settino's talk)



- <u>Phase-mixing</u> of hydromagnetic waves (small-scale generation).



(Pucci et al., JGR, 2016)

2. Exact stationary solutions for a shear flow

- **The HVM equations:**

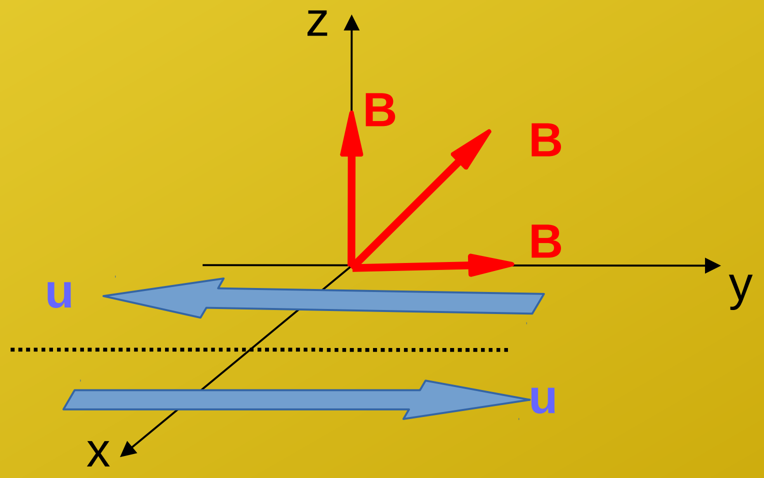
$$\frac{\partial f}{\partial t} + \mathbf{v}\cdot\nabla f + \frac{e}{m_p}\left(\mathbf{E} + \frac{\mathbf{v}}{c}\times\mathbf{B}\right)\cdot\frac{\partial f}{\partial \mathbf{v}} = 0,$$

$$\frac{\partial \mathbf{B}}{\partial t} = -c\nabla\times\mathbf{E};\quad \mathbf{j} = \frac{c}{4\pi}\nabla\times\mathbf{B},$$

$$\mathbf{E} = -\frac{1}{c}(\mathbf{u}\times\mathbf{B}) + \frac{1}{en}\left(\frac{\mathbf{j}\times\mathbf{B}}{c}\right) - \frac{1}{en}\nabla p_e,$$

Electrons: either isothermal $\boxed{p_e(x) = k_B\, n(x)\, T_e}$, ***or***, non-isothermal $\boxed{p_e(x) = k_B\, n(x)\, T_e(x)}$

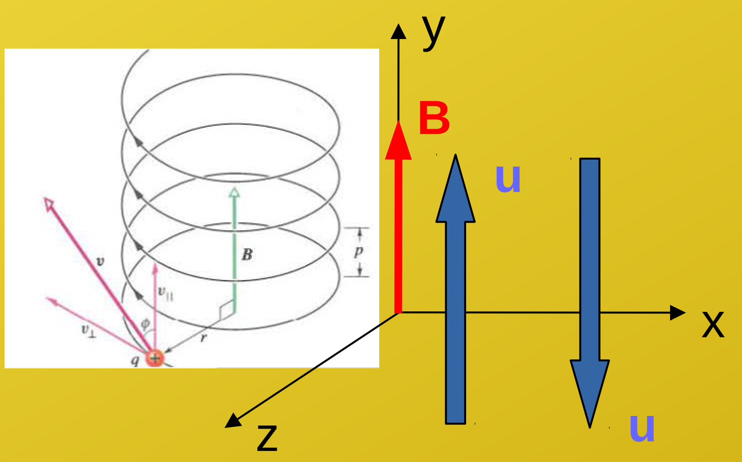
The geometry:
- sheared **u**;
- uniform **B**, with various orientations.



2.1 B parallel to u

Particle motion:
- **B** uniform
- **E** = 0

⇒ **helical motion**



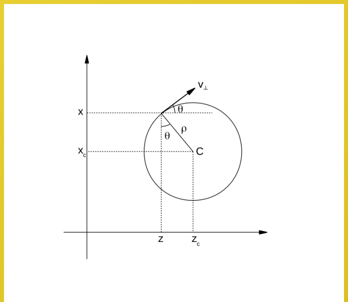
<u>Constants of motion:</u>

$$k_1 = P_y = m_p v_y$$
$$k_2 = -\frac{P_z c}{e B_0} = x - \frac{v_z}{\Omega_{cp}}$$
$$k_3 = E = \frac{m_p}{2}\left(v_x^2 + v_y^2 + v_z^2\right).$$

Ion Distribution Function: a “shifted Maxwellian <u>for guiding centers</u>”.

$$f_{eq}(x, \mathbf{v}) = \frac{n_0}{(2\pi)^{\frac{3}{2}} v_{th,i}^3} \exp\left\{-\frac{1}{2v_{th,i}^2}\left[v_x^2 + \left(v_y - U\left(x - \frac{v_z}{\Omega_i}\right)\right)^2 + v_z^2\right]\right\}$$

U(·) = guiding center velocity profile
determines the shear profile



k_2 = guiding center x-position

f(x,**v**) expressed in terms of constants of motion
⇒ **<u>stationary solution</u> of the Vlasov equation**

- Far from the shear layer (*U* = const)

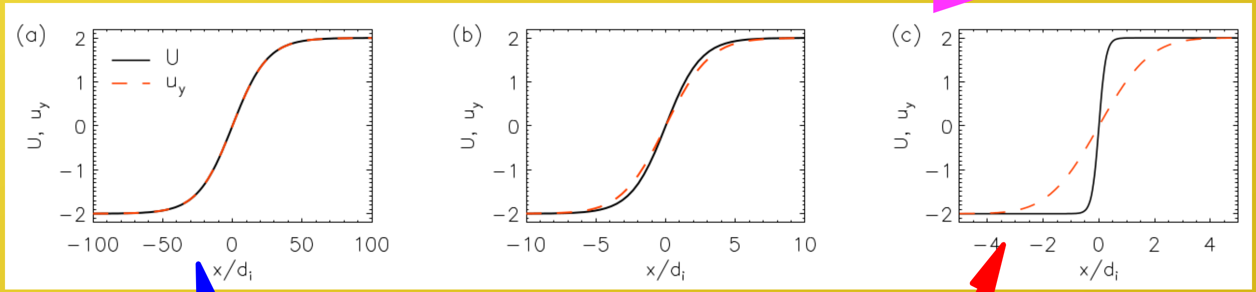
⇒ f_{eq} reduces to a shifted Maxwellian.

Moments of the Distribution Function - I

a) **density**: $n(x) = n_0$ = uniform

b) **bulk velocity** $\mathbf{u}(x)$:
- aligned with **B**;
- determined by $U(\cdot)$.

decreasing Δx



Large-scale shear ($\Delta x >> R_L$): $u_y(x) = U(x)$

Small-scale shear: $\min\{\Delta x\} \approx R_L$

- gyromotion mixes up particles at scale R_L

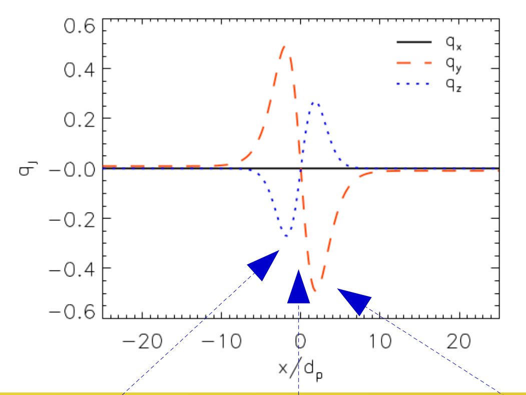
- **u** parallel to **B**;
- **j** = 0 ;

Ohm's law $\Rightarrow$ **E** = 0 $\Rightarrow$ **B** = stationary

- assuming isothermal electrons ...

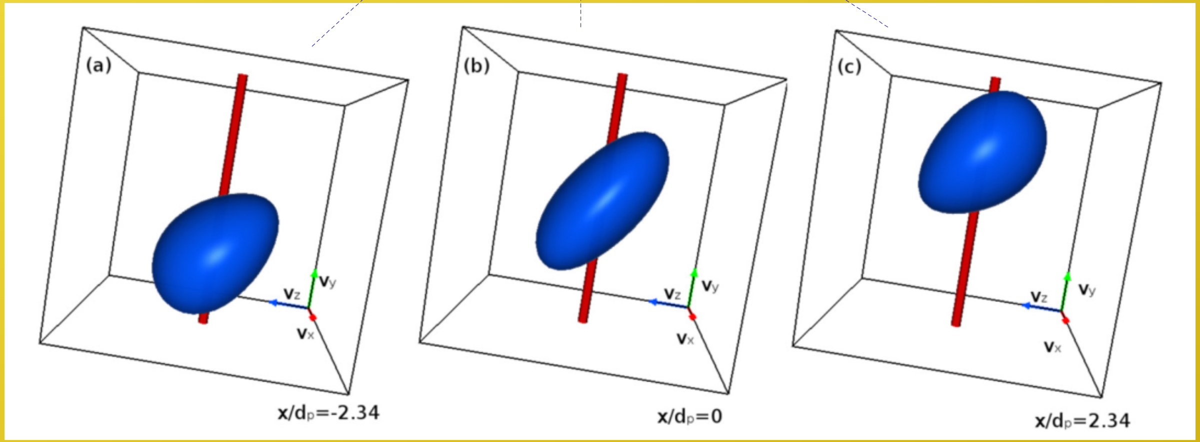
$\Rightarrow$ a stationary solution of the whole set of HVM equations

Moments of the Distribution Function - II



c) **heat flux** ϕ(x):

- localized at the shear layer;
- perpendicularly to the shear direction.

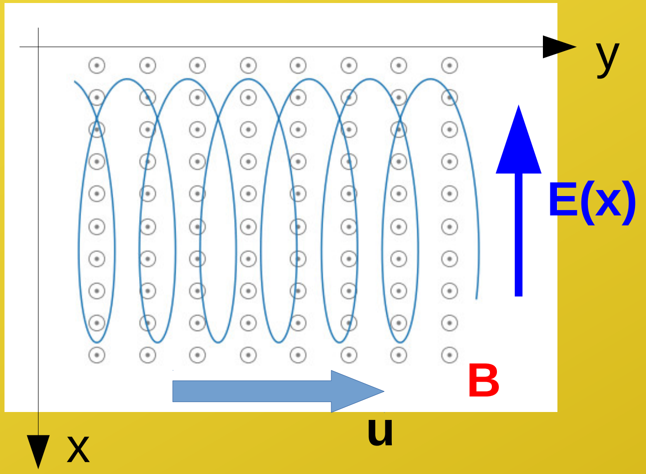


- ***temperature anisotropy;***
- ***agirotropy around B.***

Departures from maxwellianity

2.2 B perpendicular to u

- Bulk motion induced by **E** : **E** x **B** drift
- Shear flows requires ***nonuniform*** **E** = **E**(x)



Single particle motion: **a nonlinear oscillator**

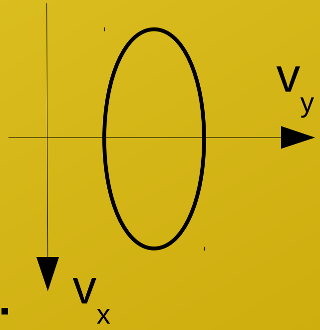
$$v_y = -\Omega_{cp} x + W_0,$$

$$\frac{d^2x}{dt^2} = -\Omega_{cp}^2 x + \frac{e}{m_p} E(x) + \Omega_{cp} W_0,$$

- particles trapped inside a potential well ⇒

$$U_{eff}(x) = e\phi(x) + \frac{1}{2} m_p \Omega_{cp}^2 \left(x - \frac{W_0}{\Omega_{cp}} \right)^2$$

in the v_x-v_y plane:
closed trajectory and **periodic motion.**



Therefore, we can define the constants of motion:

a) ***guiding center position***: $x_{gc} = \langle x \rangle_t$

b) ***guiding center velocity***: $v_{gc} = \langle v_y \rangle_t$

c) ***drift kinetic energy***: $\mathscr{E}_{drift} = \frac{1}{2} m_p v_{gc}^2$

The Distribution Function

$$f(x,\mathbf{v}) = C\exp\left\{\frac{1}{\kappa_B T_0}\left[\frac{m_p(v_x^2+v_y^2+v_z^2)}{2} + e\phi(x;v_x,v_y) - \mathscr{E}_{drift}(x,v_x,v_y)\right]\right\}$$

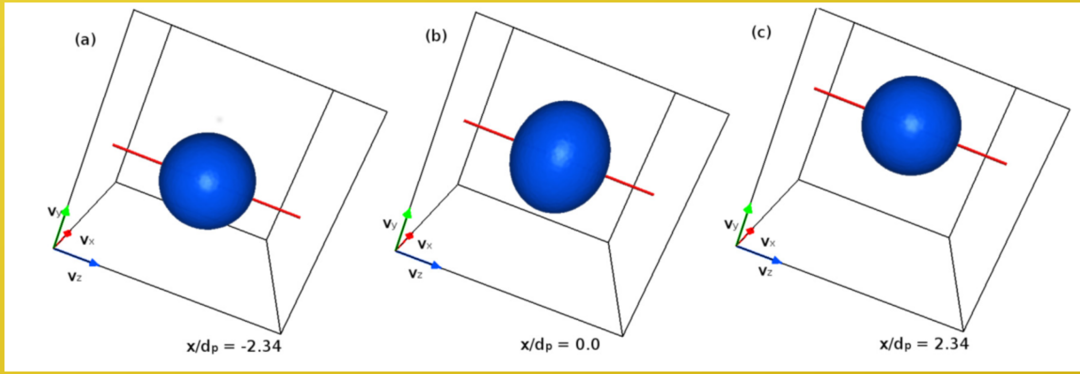
- ϕ = electric potential, such that $\phi(x_{gc}(x,\mathbf{v}),\mathbf{v}) = 0$;
- $\mathcal{E}_{drift}$ is subtracted to total energy;
- $f(x,\mathbf{v})$ reduces to a shifted Maxwellian for $\mathbf{E}(x)$ = const .

- **p_e chosen to satisfy the Ohm's law**

⇒ non-isothermal electrons: $T_e = T_e(x)$

The DF is implicitly defined
⇒ it must be numerically calculated solving particle orbits

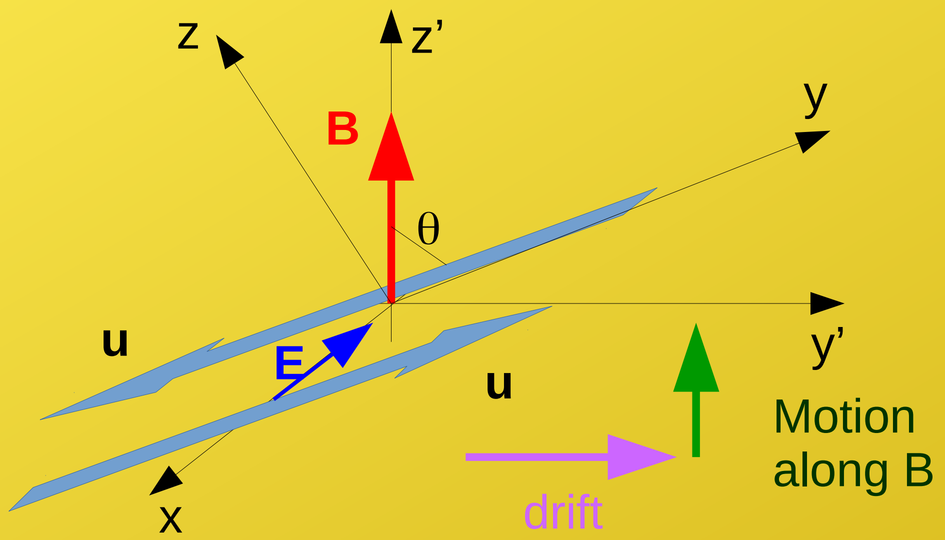
1) a grid in the 3D phase space $\{x,v_x,v_y\}$ is set
2) for **each grid point**:
   a) the particle trajectory is integrated along one orbit;
   b) the guiding center velocity v_{gc} and position x_{gc} are calculated.



Results

- Bulk velocity: $\mathbf{u}(x) \sim c\,\dfrac{\mathbf{E}(x)\times\mathbf{B}}{B^2}$
- Anisotropic temperature;
- DF gyrotropic around **B.**

2.3 oblique B *(still preliminar...)*



A trick: rotating the reference frame

Single particle dynamics:
drift motion + motion along B.

Parallel guiding center velocity:

$$v_{||,gc}(x,\mathbf{v}) = \frac{v_{\perp,gc}(x,\mathbf{v})}{\tan\theta}$$

(constant of motion)

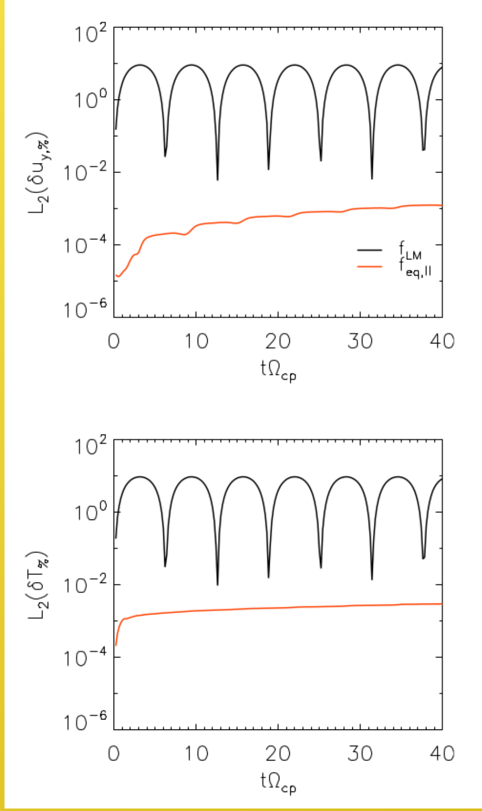
A stationary Distribution Function

$$f(x,\mathbf{v}) = C\exp\left\{\frac{1}{\kappa_B T_0}\left[\frac{m_p\left[v_x^2+v_{y'}^2+(v_{z'}-v_{||,gc}(x,\mathbf{v}))^2\right]}{2}+e\phi(x;v_x,v_y)-\mathscr{E}_{drift}(x,v_x,v_y)\right]\right\}$$

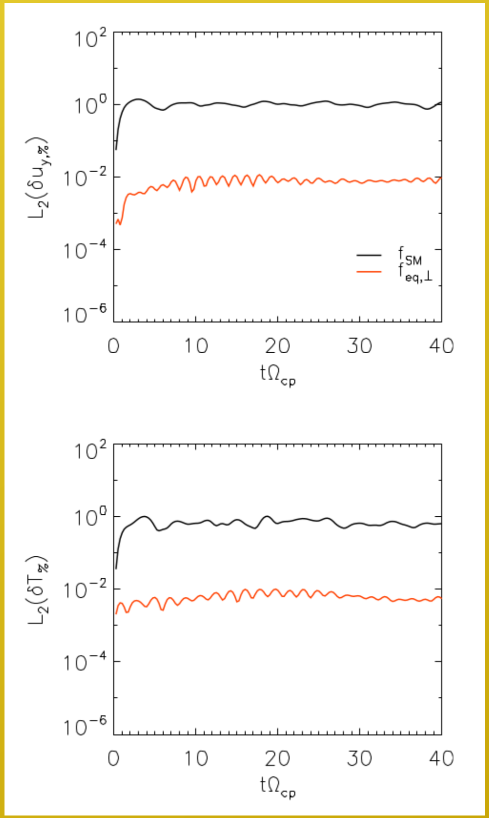
3. Numerical simulations

Stationarity of solutions checked through the **HVM numerical code**

B *parallel to* ***u***



B *perpendicular to* ***u***



Percentage departures $\boldsymbol{L_2}(\delta u_{y\%})$, $\boldsymbol{L_2}(\delta T_{\%})$ from initial state, vs time

u_y component

red: exact solutions
black: local Maxwellian

temperature

Exact solution:

- departures δu_y, $\delta T \approx 10^{-5} - 10^{-4}$
- 2 - 3 orders of magnitude lower than for shifted maxwellian

4. Conclusions

- Stationary solutions of HVM equations representing magnetized shear flows.
- B parallel to u(x): analytical DF; ion temperature anisotropy and agyrotropy at the shear layer; isothermal electrons.
- B perpendicular to u(x): numerical DF; ion temperature anysotropy; adiabatic electrons.
- B oblique: numerical DF (preliminar).
- Stationarity checked through numerical simulations (HVM code).

Thank you !