

# Minimal self-consistent dark matter phase-space distribution functions confronted to simulations

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!nterTalentum



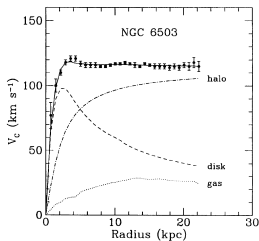
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Teórica  
UAM-CSIC

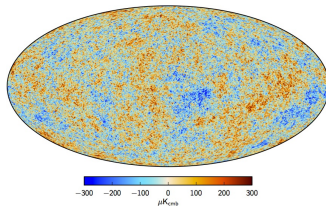
# The dark matter problem

## Galactic rotation curves



Begeman+ 1991

## CMB anisotropies



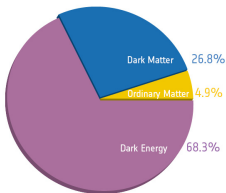
Planck Collaboration 2015

## Gravitational lensing



Credit: NASA/ESA

## Success of $\Lambda$ CDM



Credit: ESA/Planck

# Phase-space distribution of DM & theoretical uncertainties for direct and indirect searches

## Direct searches

$$\frac{dR}{dE} \propto \rho_{\odot} \int_{v_{\min} \leq |\vec{v}| \leq v_{\text{esc}}} \frac{f_{\odot}(\vec{v})}{|\vec{v}|} d^3v$$

Impact at low masses

$$v_{\min} \sim v_{\text{esc}}$$

## Speed-dependent annihilation

- $\langle \sigma v \rangle(r) \propto \langle v_r^2 \rangle$  ***p*-wave**  
 $\sigma v = \sigma_1 v_r^2 \Rightarrow \langle \sigma v \rangle \equiv \langle \sigma v \rangle(r)$

$$\langle \sigma v \rangle(r) = \sigma_1 \int d^3v_1 d^3v_2 f_r(\vec{v}_1) f_r(\vec{v}_2) v_r^2$$

- $\langle \sigma v \rangle(r) \propto \langle 1/v_r \rangle$  or  $\langle 1/v_r^2 \rangle$

**Sommerfeld enhancement**

## Primordial black holes

- Gravitational microlensing event rates (EROS, MACHO)

$$\frac{d\Gamma}{dt} \propto \rho(r) \int v f(\vec{v}, \vec{r}) d^3v$$

- Merger rates (gravitational waves)

+ DM subhalos (test masses), disruption of stellar binaries

# Standard approaches 1: "Standard halo model"

## Standard halo model (SHM)

Maxwell-Boltzmann distribution

$$f(\vec{v}) = \frac{1}{v_0^3 \pi^{3/2}} e^{-\left(\frac{\vec{v}}{v_0}\right)^2}$$

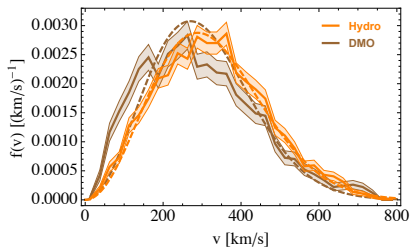
## Oversimplification

- Isothermal sphere
- Infinite system
- Ad hoc truncation at  $v_{\text{esc}}$

# Standard approaches 2: direct fits to simulations

## General insight

Generic features found in simulations (e.g., cusp/cores)



Bozorgnia+ 2017

## But insufficient approach

- Extrapolations based on fits at 8 kpc
- **Peak speed free parameter**  
⇒ not connected to circular speed
- MW one particular realization
- MW constrained system (e.g., Gaia)
- Subgrid physics

## Self-consistent approach required

- Eddington-like methods: next-to-minimal approach
- Method applied blindly to direct searches so far  
→ Timely to study validity range in detail

# Phase space of dark matter from first principles

## Phase-space distribution $f(\vec{v}, \vec{r})$ : closed Vlasov-Poisson system

- Vlasov equation, steady state (equilibrium model)

$$\{f, H\} = \vec{v} \cdot \frac{\partial f}{\partial \vec{r}} - \frac{\partial \Phi}{\partial \vec{r}} \cdot \frac{\partial f}{\partial \vec{v}} = 0$$

→ Jeans' theorem:  $f \equiv f(I_1, \dots, I_N)$  where  $\{I_i, H\} = 0$

- Poisson equation

$$\Delta \Phi = 4\pi G \rho \quad \text{with} \quad \rho = \int f(\vec{v}, \vec{r}) d^3v$$

## Eddington's inversion formula (Eddington 1916)

Isotropic system + spherical symmetry:  $f(\vec{v}, r) \equiv f(\mathcal{E})$

with  $\mathcal{E} = \Psi(r) - \frac{v^2}{2}$  and  $\Psi(r) = \Phi(R_{\max}) - \Phi(r)$

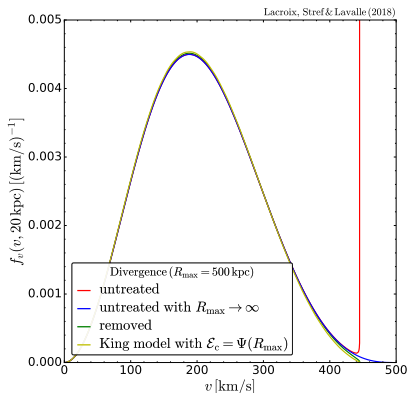
$$f(\mathcal{E}) = \frac{1}{\sqrt{8}\pi^2} \left[ \frac{1}{\sqrt{\mathcal{E}}} \left( \frac{d\rho}{d\Psi} \right)_{\Psi=0} + \int_0^{\mathcal{E}} \frac{d^2\rho}{d\Psi^2} \frac{d\Psi}{\sqrt{\mathcal{E} - \Psi}} \right]$$

+ anisotropic extensions

# Theoretical consistency and radial boundary

## Imposing a radial boundary

- Finite system ( $R_{\max}$ )  $\Rightarrow$  divergence of  $f(\vec{r}, \vec{v})$  at  $v_{\text{esc}}$  (from  $1/\sqrt{\mathcal{E}}$ )
- $v_{\text{esc}}$  crucial (direct DM searches at low masses, stellar surveys)
- Regularization: modified profile or energy cutoff (King)



Velocity distribution:

$$f_{\vec{v}}(\vec{v}, r) \equiv \frac{f(\mathcal{E}, L)}{\rho_{\text{DM}}(r)}$$

Speed distribution ( $v = |\vec{v}|$ ):

$$f_v(v, r) \equiv v^2 \int d\Omega_v f_{\vec{v}}(\vec{v}, r)$$

For isotropic distribution:

$$f_v(v, r) = \frac{4\pi v^2}{\rho_{\text{DM}}(r)} f\left(\Psi(r) - \frac{v^2}{2}\right)$$

# Theoretical consistency: instabilities

## Validity range of the method

- Standard criterion:

$$f \geq 0$$

- Antonov instabilities for some DM-baryon configurations

- Stable solution if

$$\frac{df}{d\mathcal{E}} > 0 \Leftrightarrow \frac{d^2\rho}{d\Psi^2} > 0$$

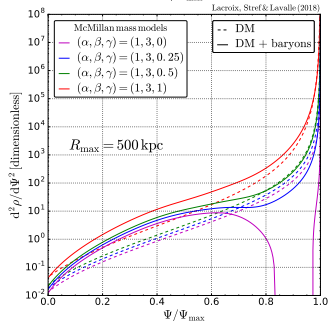
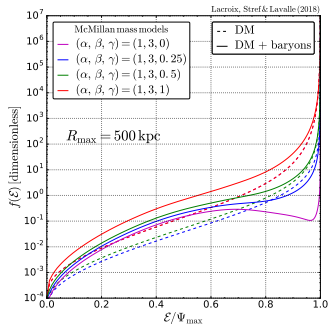
Doremus+ 1971, Kandrup & Sygnet 1985

- Select mass models

Lacroix+ 2018

For anisotropic systems criteria against radial perturbations only

Doremus+ 1973





# Actual predictivity of Eddington's formalism?

## Tests with cosmological simulations

### Description

- 2 sets of simulations

Mollitor+ 2015

$M_{\text{DM}} = 2.3 \times 10^5 M_{\odot}$ ,  $H_{\text{sm1}} = 150$  pc

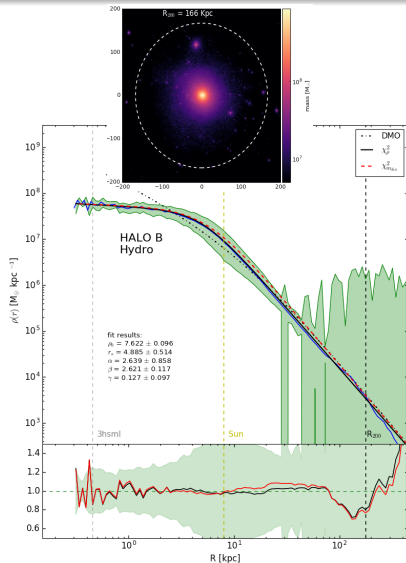
Núñez+ 2019, in prep.

$M_{\text{DM}} = 1.9 \times 10^5 M_{\odot}$ ,  $H_{\text{sm1}} = 35$  pc

- 20 Mpc boxes + zoom-in
- DM-only + hydro

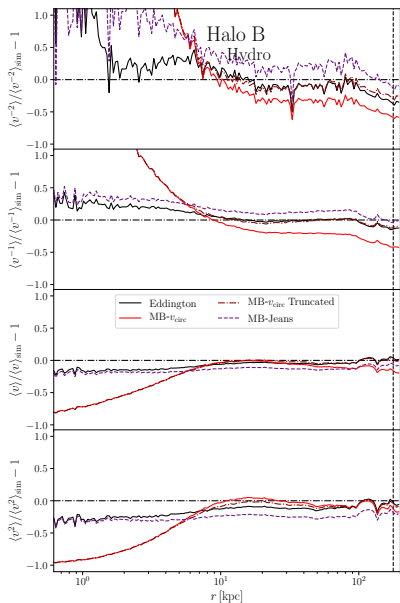
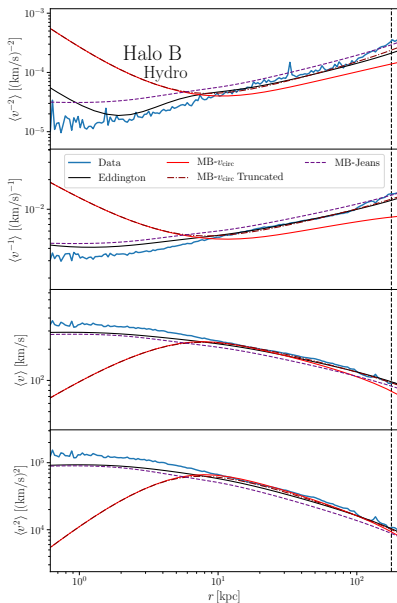
### Procedure

- Fit mass model from simulation  
 $\Rightarrow \rho_{\text{DM}}, \rho_{\text{B}}, \Psi = \Psi_{\text{DM}} + \Psi_{\text{B}}$
- Input for Eddington's method
- Comparison with simulation outputs



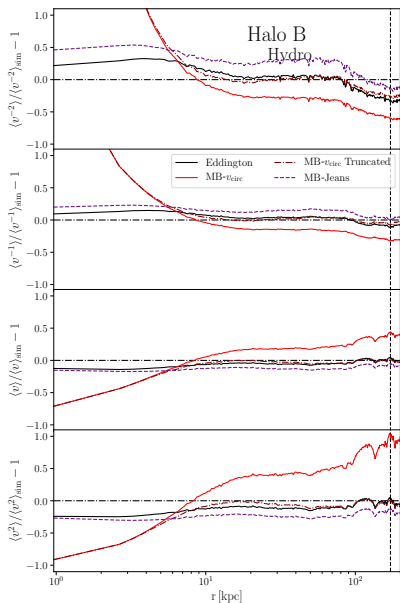
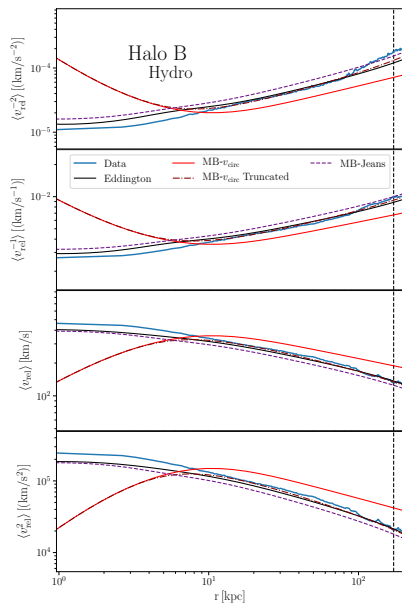
Lacroix+ 2019, in prep.

# Moments of the speed distribution



Lacroix+ 2019, in prep.

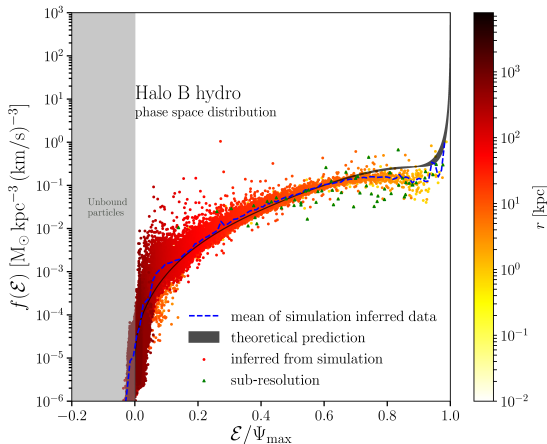
# Moments of the *relative* speed distribution



# Ergodic phase-space distribution

Reconstruction from 2D bins  $ij$  in phase space ( $r_i, v_j$ )

$$f(\mathcal{E}) = m \frac{d^6 N}{d^3 x d^3 v} \rightarrow f(\mathcal{E})_{ij} = \frac{m}{(4\pi r_i v_j)^2} \frac{N_{ij}}{\Delta r_i \Delta v_j}$$



$\Rightarrow$  consistency check

Lacroix+ 2019, in prep.

# Summary

## Eddington's inversion method

- A few physical assumptions, moderate level of technicalities
- Self-consistent solution of Vlasov-Poisson system
- Mass model direct input
- Better control astrophysical uncertainties for DM searches
- Readily applicable to variety of objects

## Self-consistency: theoretical validity range

- Radial boundary (direct searches)
- Positive DF + stability

## Actual predictivity?

- Testing the method against cosmological simulations
- Not direct fits!!!
- Eddington method gives a predictivity at  $O(10\%)$

Lacroix+ 2019, in prep.

Thank you for your attention!



# Backup slides



# Application: constraints on sub-GeV DM from cosmic positrons - $p$ -wave annihilation

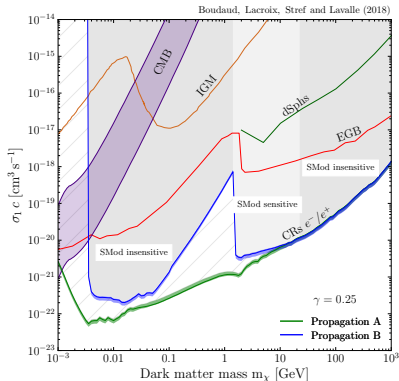
## $p$ -wave annihilation

$$\sigma v = \sigma_1 v_{\text{r}}^2 \Rightarrow \langle \sigma v \rangle \equiv \langle \sigma v \rangle(r)$$

$$\langle \sigma v \rangle(r) = \sigma_1 \int d^3 v_1 d^3 v_2 f_{\text{r}}(\vec{v}_1) f_{\text{r}}(\vec{v}_2) v_{\text{r}}^2$$

$$\Rightarrow \psi_{\text{e}} \neq \langle \sigma v \rangle \int \rho^2(r) d^3 r$$

- Very strong  $e^+$  constraints
- Justifies focusing on Eddington's methods
- Robust w.r.t. uncertainties on anisotropy and propagation

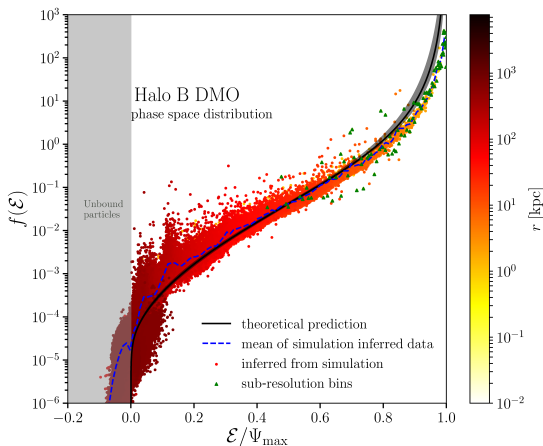


Boudaud+ 2018

# Phase-space distribution - DM only

Reconstruction from 2D bins  $ij$  in phase space ( $r_i, v_j$ )

$$f(\mathcal{E}) = m \frac{d^6 N}{d^3 x d^3 v} \rightarrow f(\mathcal{E})_{ij} = \frac{m}{(4\pi r_i v_j)^2} \frac{N_{ij}}{\Delta r_i \Delta v_j}$$



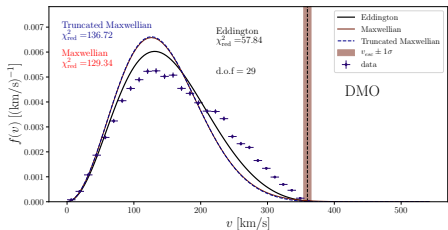
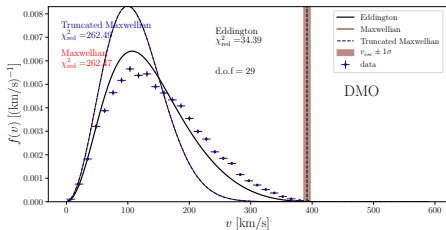
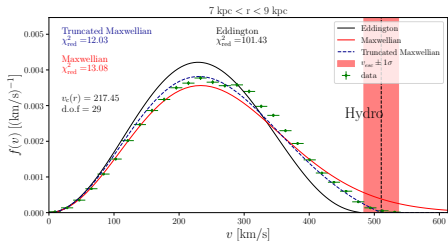
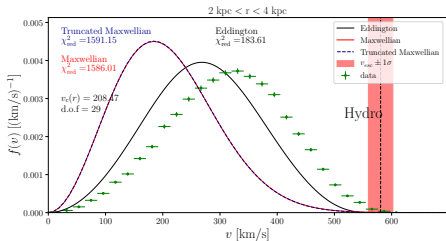
$\Rightarrow$  consistency check

Lacroix+ 2019, in prep.

# Speed distribution $f_v(v, r)$

$r = 3$  kpc

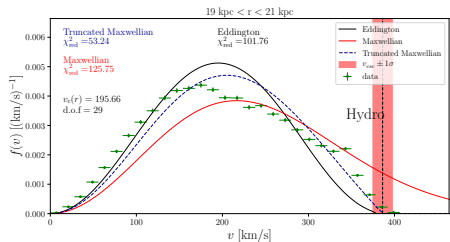
$r = 8$  kpc



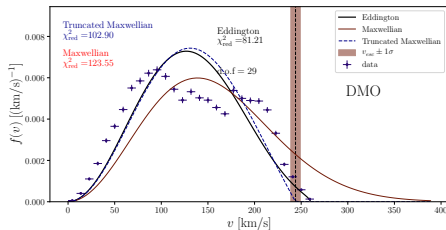
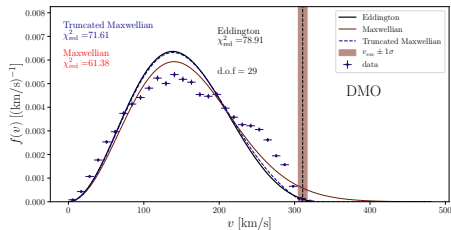
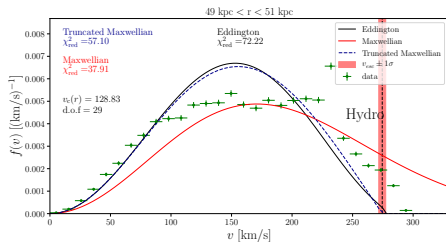
Lacroix+ 2019, in prep.

# Speed distribution $f_v(v, r)$

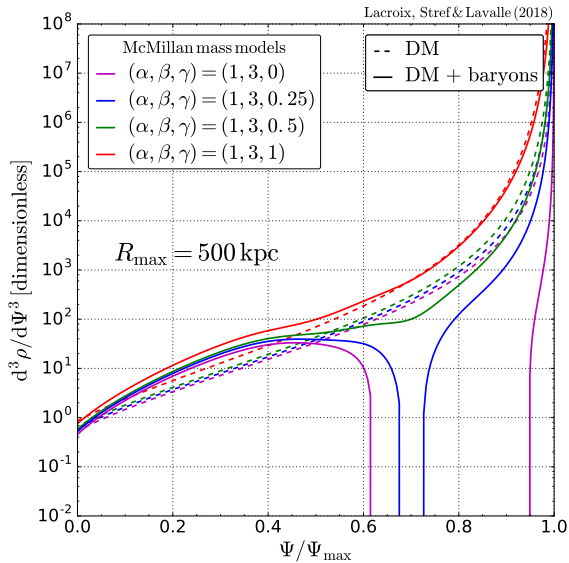
$r = 20$  kpc



$r = 50$  kpc



Lacroix+ 2019, in prep.

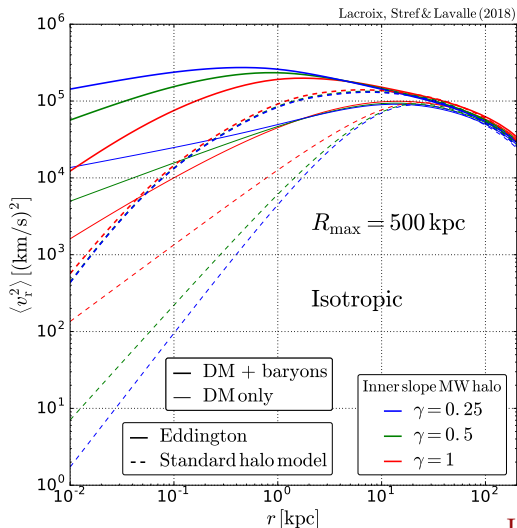


Lacroix+ 2018

# Self-consistent predictions for indirect DM searches

Prototypical case: p-wave annihilation

$$\langle\sigma v\rangle(r) \propto \langle v_r^2\rangle$$

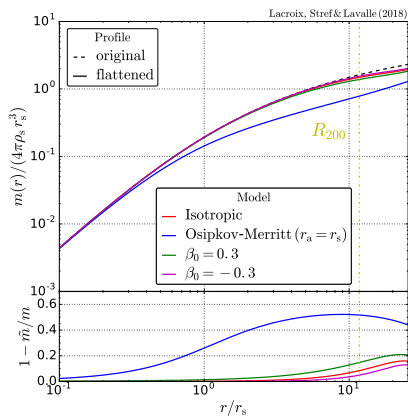
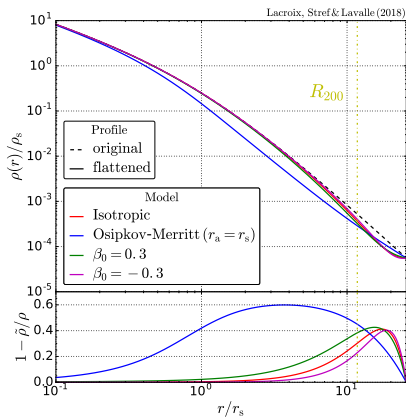


Lacroix+ 2018

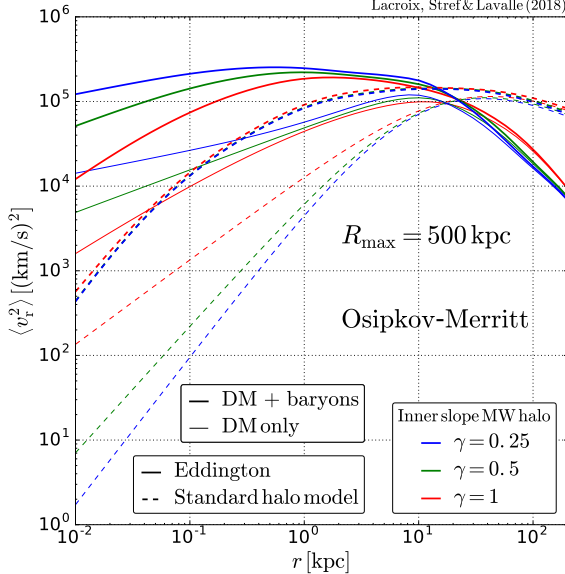
# Proper treatment critical for self-consistency

Removing divergence by hand

⇒ approach no longer self-consistent (Poisson)

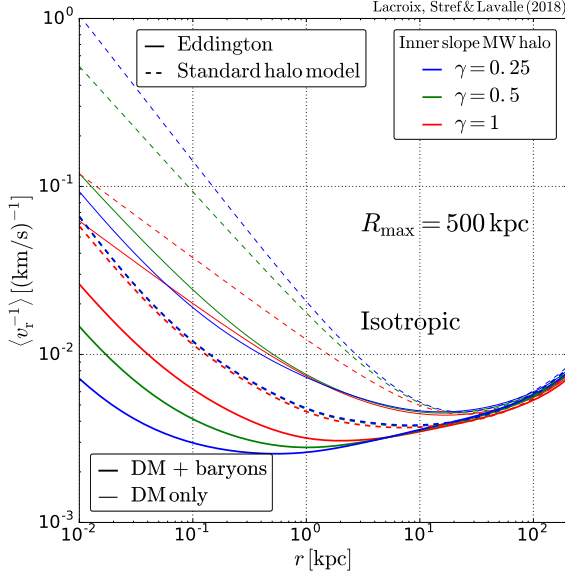


Lacroix+ 2018

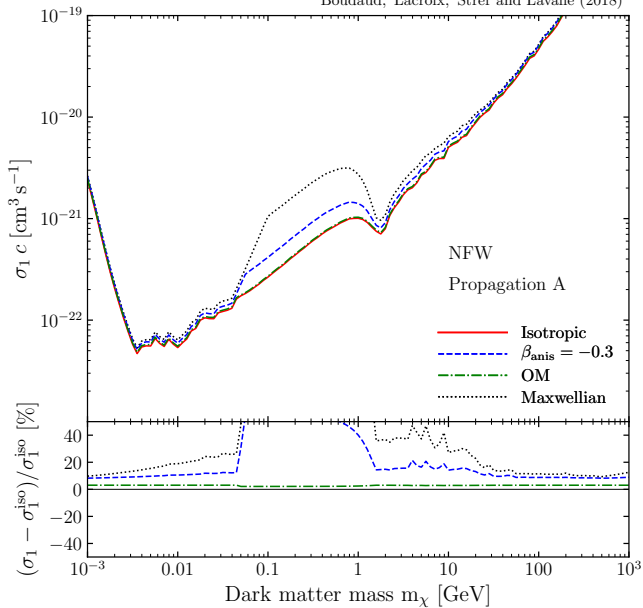


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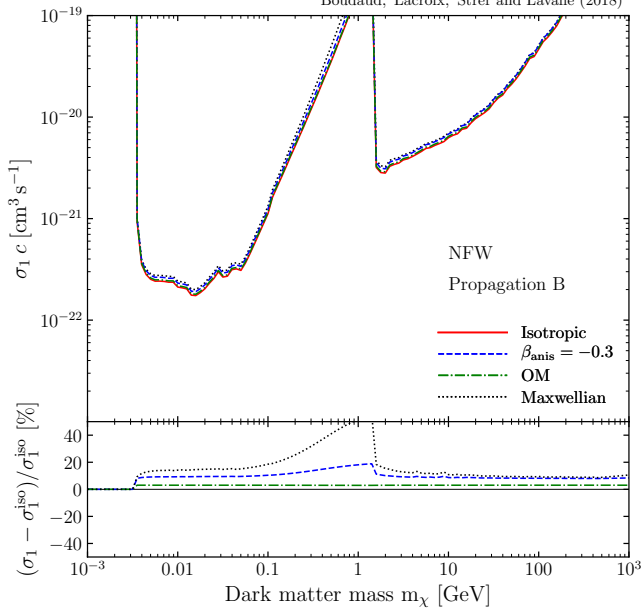




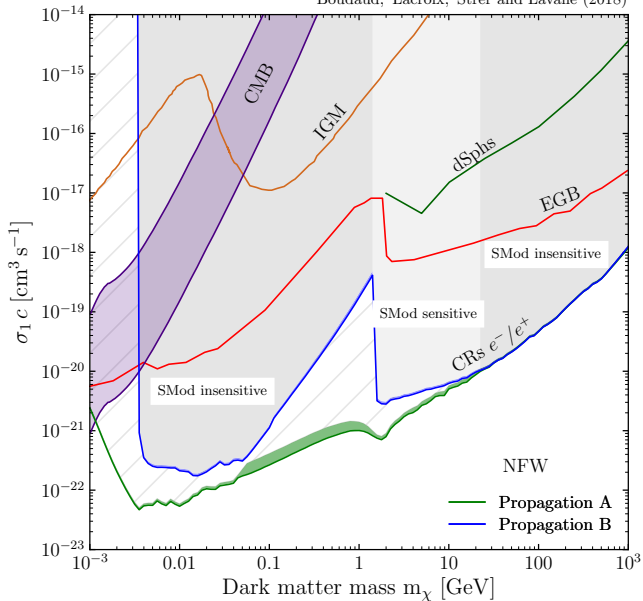
Lacroix+ 2018



Boudaud+ 2018



Boudaud+ 2018



Boudaud+ 2018

# Actual predictivity of Eddington's formalism?

## Tests with cosmological simulations

### Comparison of theoretical predictions with simulation outputs

- Speed distribution  $f_v(v, r)$
- Moments of the velocity distribution  $\langle v^n \rangle(r) = \int d^3\vec{v} v^n f_{\vec{v}}(\vec{v}, r)$
- Moments of the relative velocity distribution

$$\langle v_{\text{rel}}^n \rangle(r) = \int d^3\vec{v}_{\text{rel}} F_{\text{rel}}(\vec{v}_{\text{rel}}, r) v_{\text{rel}}^n$$

where

$$F_{\text{r}}(\vec{v}_{\text{r}}, r) = \int d^3\vec{v}_{\text{c}} f_{\vec{v}}(\vec{v}_1, r) f_{\vec{v}}(\vec{v}_2, r)$$

- Full ergodic phase-space distribution  $f(\mathcal{E})$

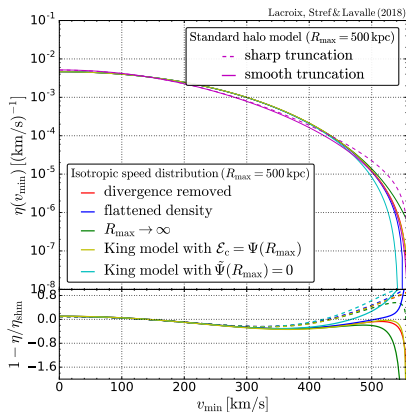
### Caveat

- Not fits to simulation results
- Simulations only used to test theoretical predictions

# Impact on predictions for direct DM searches

Event rate proportional to

$$\eta(v_{\min}) = \int_{v_{\min} \leq v \leq v_{\oplus} + v_{\text{esc}}} \frac{f_{\vec{v}, \oplus}(\vec{v})}{v} d^3v$$



Lacroix+ 2018