Fluctuation and relaxation in globular clusters

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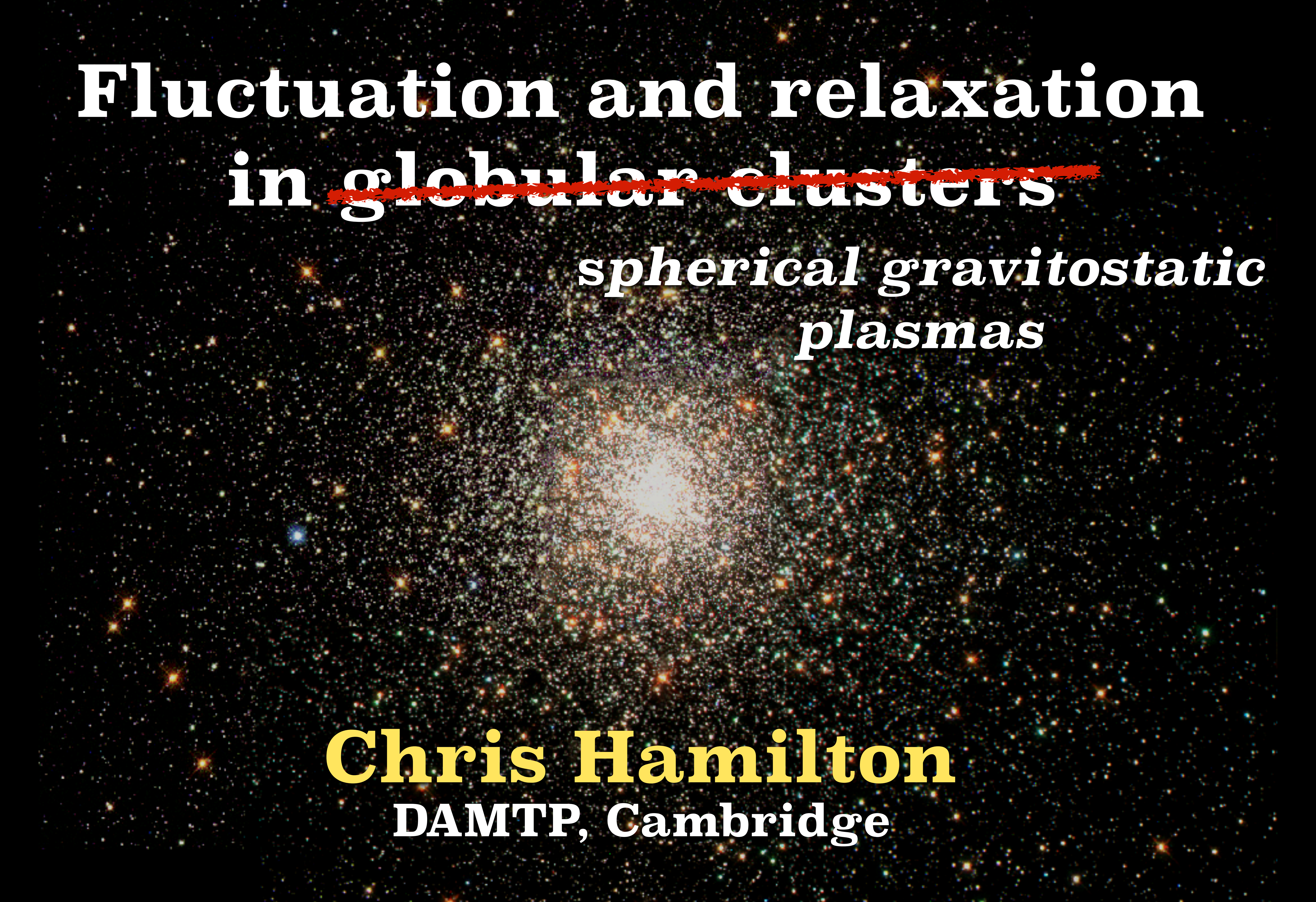
Hamilton, Fouvry, Binney, Pichon (2018) *MNRAS* | @ VLASOVIA, 24th July 2019



Fluctuation and relaxation in ~~globular clusters~~

spherical gravitostatic plasmas

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Globular cluster properties

- Dense, spherical stellar systems (with no central SMBH!)
- Radii ~ a few parsecs
- Consist of ~10^5 similar stars
- Very old (~10^{10} yr)
- Crossing time ~10^5 yr
- Relaxation time ~10^{10} yr

Q: what physics drives cluster evolution over very long (> Gyr) timescales?

N-body problem

Clusters well approximated as clean N-body systems: point masses interacting via Newtonian gravity

$$\mathbf{F}_i = -\sum_{j \neq i} \frac{G m_i m_j}{|\mathbf{x}_i - \mathbf{x}_j|^3} (\mathbf{x}_i - \mathbf{x}_j)$$

$$i, j = \{1, ..., N\}$$

($N \sim 10^5$)

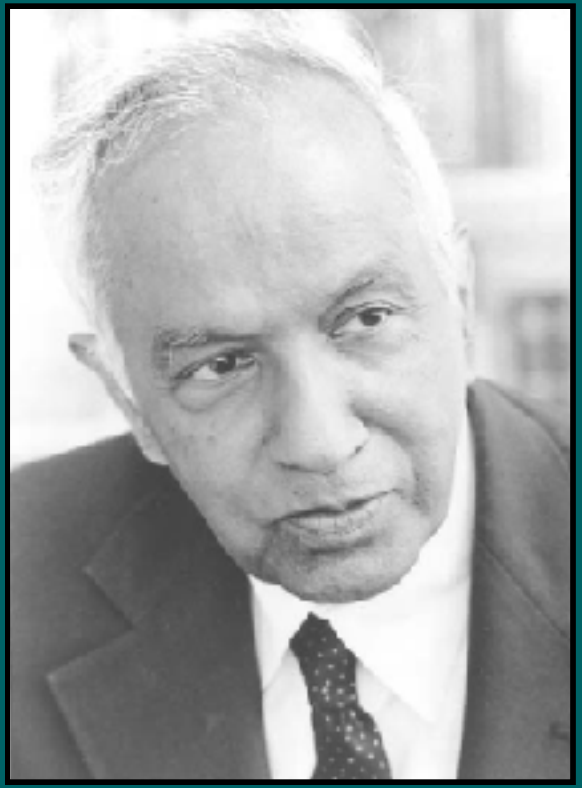
so we could just code up 10^5 equations of motion: problem solved?

N-body integration

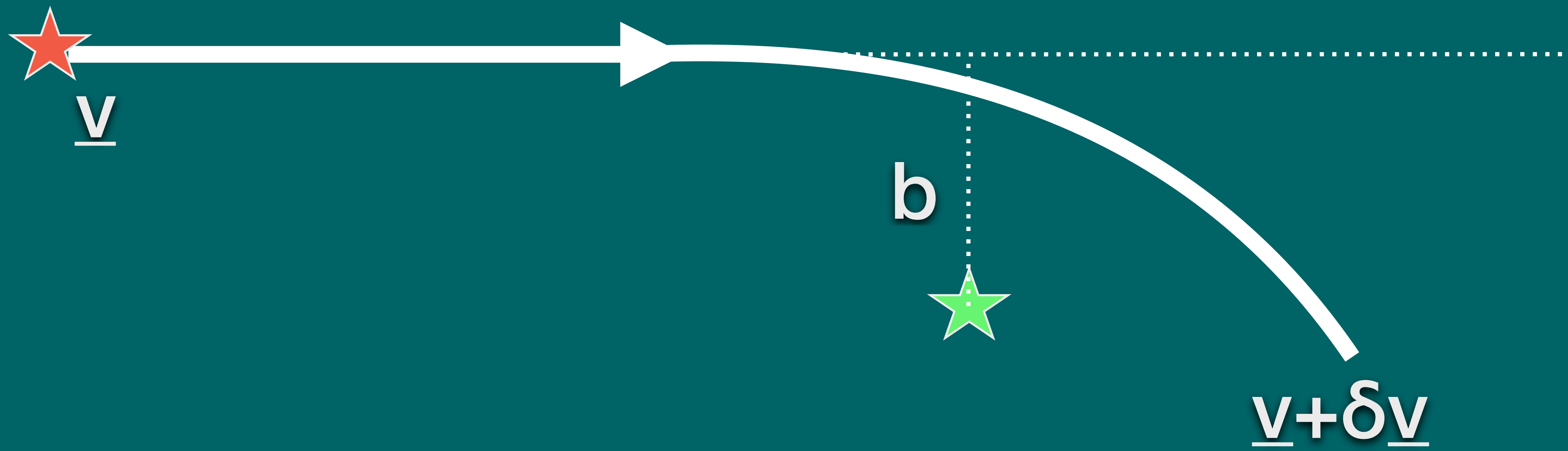
- is very expensive: it has only recently become feasible for N~10^5.
- doesn't provide much physical insight.

Instead, people have mostly relied on Chandrasekhar's theory of two body relaxation.

Chandrasekhar's theory of two-body relaxation



- replace cluster with an infinite, homogeneous, fixed background field of stars
- a test star moves in a straight line except during an encounter with a field star with impact parameter b



- By summing up many independent two-body encounters we get drift and diffusion coefficients

drift
$$\langle \delta \mathbf{v} \rangle_{\mathrm{f}} = 4\pi G(m+m') \log \Lambda \frac{\partial h}{\partial \mathbf{v}}$$

diffusion:
$$\langle \delta v_i \delta v_j \rangle_{\mathrm{f}} = 4\pi G m' \log \Lambda \frac{\partial^2 g}{\partial v_i \partial v_j}$$

which can be inserted into a homogeneous Fokker-Planck equation.

- Both coefficients involve the Coulomb logarithm:

$$\Lambda \equiv \frac{b_{\max}}{b_{\min}}$$

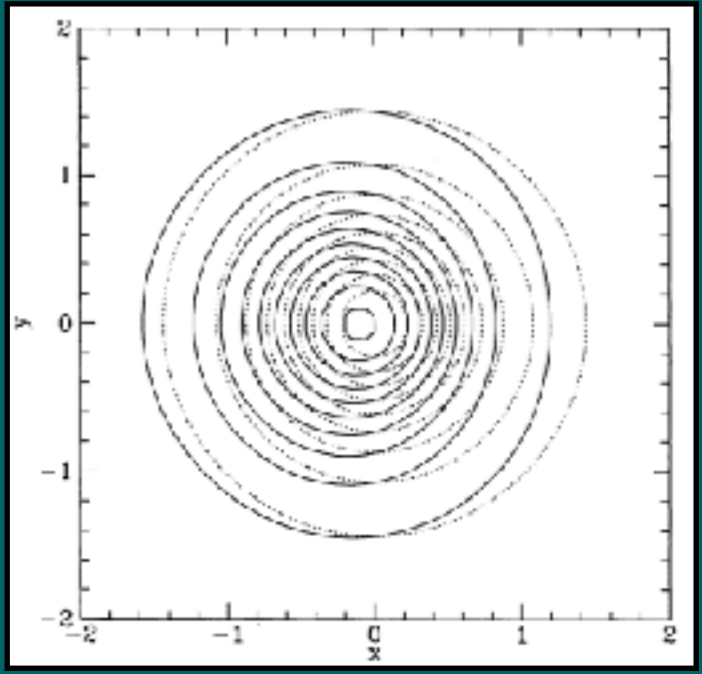
Chandrasekhar's theory has been the standard picture of cluster evolution for several decades!

Chandrasekhar's false assumptions

- unlike plasmas, stellar systems are not homogeneous! (no Debye screening)
- individual unperturbed orbits are complicated (not straight lines)
- Coulomb logarithm log Λ is essentially a free parameter $$\Lambda \equiv \frac{b_{\rm max}}{b_{\rm min}}$$
- 'field stars' are not inert: the system should evolve self-consistently $$\nabla^2\Phi = 4\pi G \int {\rm d}^3\mathbf{v}\, f$$
- Chandrasekhar ignores collective effects (self-gravitating oscillations of entire system)

Collective effects (self-gravity)

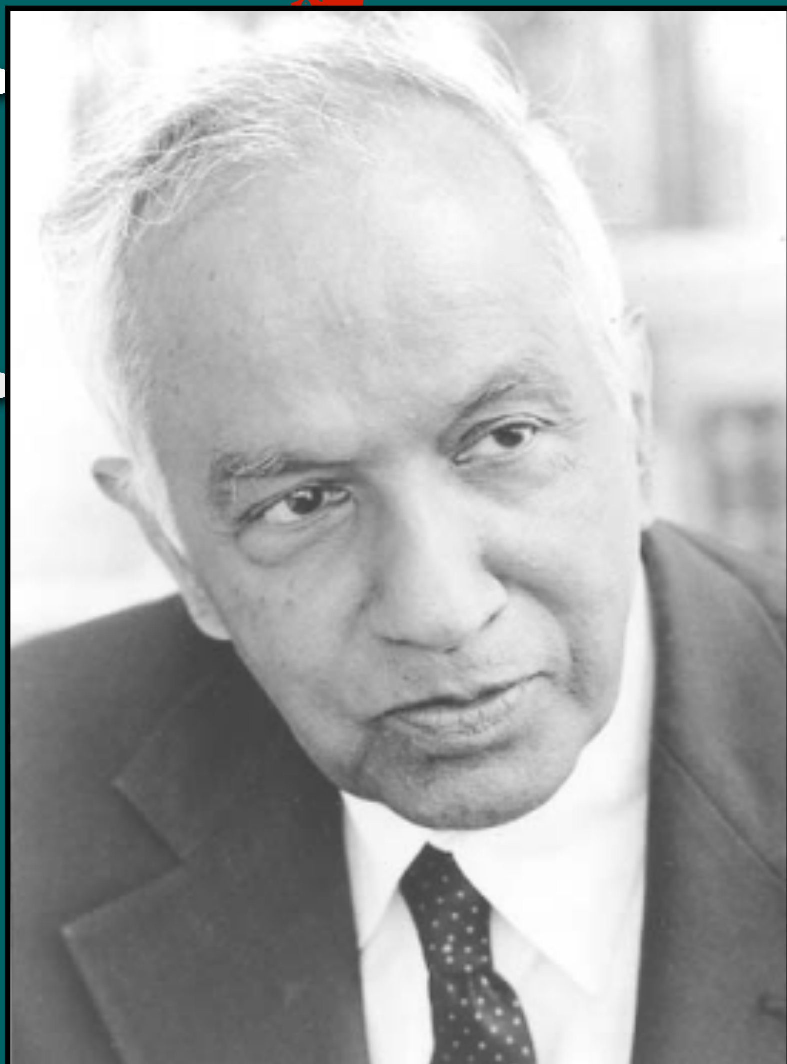
- Electrostatic plasmas have natural oscillation modes: their effect is captured by a dielectric function
- stellar systems also have normal modes: coherent (Vlasov-stable) oscillations of whole cluster



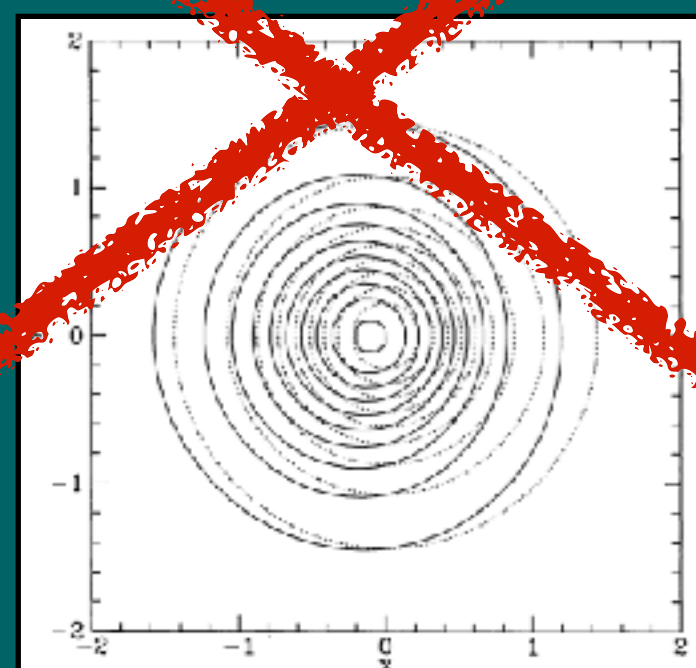
Example of the ℓ = 1 (dipole) sloshing mode in a globular cluster (Weinberg 1993)

- so a star cluster is a polarisable medium and has a "dielectric function" that we can calculate

Collective effects (self-gravity)



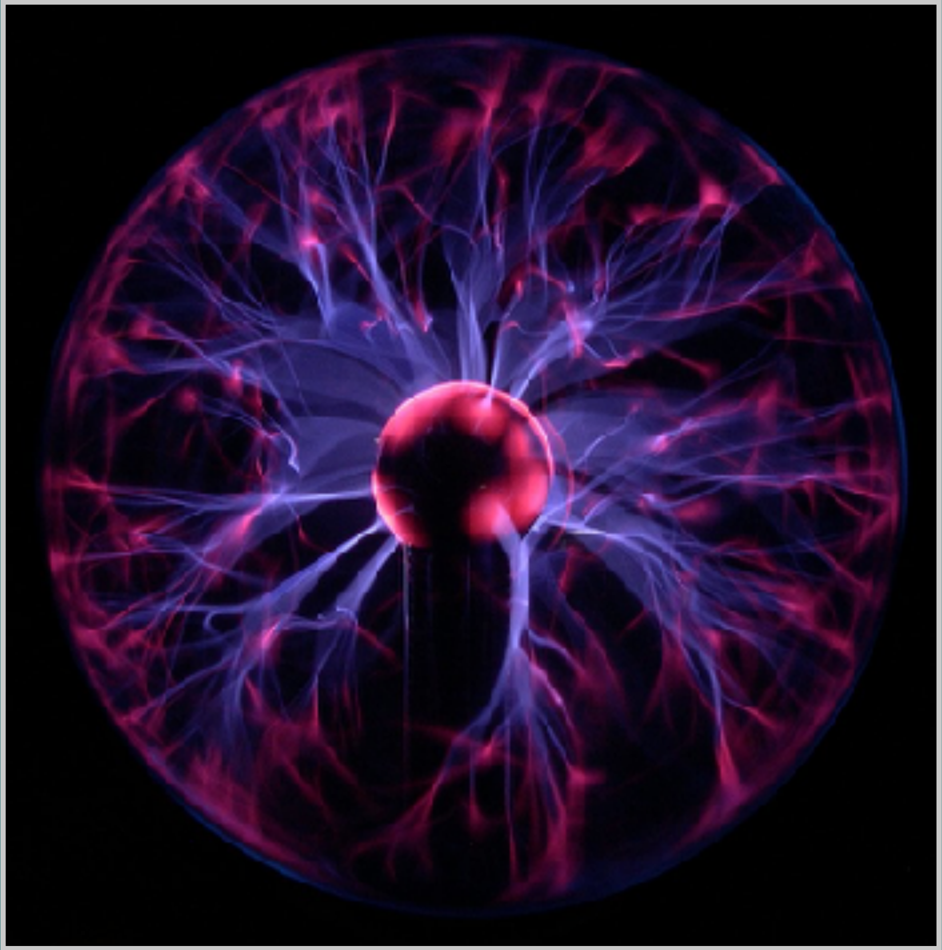
- …smas have **natural oscillation modes**: …ptured by a **dielectric function**
- …also have normal modes: coherent …oscillations of whole cluster



Example of the ℓ=1 (dipole) mode in a globular cluster (Weinberg 1993)

- so a star cluster is a **polarisable medium** and has a "dielectric function" that we can calculate

A modern alternative: the Balescu-Lenard equation



Balescu (1960)
Lenard (1960)

Heyvaerts (2010)
Chavanis (2012)

Balescu-Lenard (BL): a self-consistent mean field theory

Boltzmann + Poisson:

$$\frac{\partial f}{\partial t} + [f, H] = 0.$$

$$\nabla^2 \Phi = 4\pi G \int \mathrm{d}^3\mathbf{v}\, f$$

- Solve with mean field + Poisson fluctuations ('noise')
- Use angle-action variables to account for inhomogeneity

$$H(\boldsymbol{\theta}, \mathbf{J}, t) = H_0(\mathbf{J}) + \Phi_1(\boldsymbol{\theta}, \mathbf{J}, t),$$

$$f(\boldsymbol{\theta}, \mathbf{J}, t) = f_0(\mathbf{J}) + f_1(\boldsymbol{\theta}, \mathbf{J}, t).$$

J = (J_r,L) = (radial action, orbital angular momentum)

The **BL equation** provides a **new physical picture** of cluster relaxation, driven by inherent **correlated noise**:

$$\frac{\partial f_0}{\partial t} = \langle [f_1, \Phi_1] \rangle = -\frac{\partial}{\partial \mathbf{J}} \cdot \mathbf{F}$$

correlated self-consistent fluctuations due to √N noise

inhomogeneity

resonant interactions determined by orbital frequencies

$$\mathbf{F} = \sum_{\mathbf{k}\mathbf{k}'} \mathbf{k} \int \mathrm{d}\mathbf{J}' \frac{\delta(\mathbf{k}\cdot\mathbf{\Omega} - \mathbf{k}'\cdot\mathbf{\Omega}')}{|\epsilon_{\mathbf{k}\mathbf{k}'}(\mathbf{J},\mathbf{J}',\mathbf{k}\cdot\mathbf{\Omega})|^2} \left(\mathbf{k}\cdot\frac{\partial}{\partial \mathbf{J}} - \mathbf{k}'\cdot\frac{\partial}{\partial \mathbf{J}'}\right) f_0(\mathbf{J}) f_0(\mathbf{J}')$$

(‘wavevectors’ k_i are integers)

‘dielectric function’ encoding collective effects

pros

- No cutoffs required at large impact parameters (no Coulomb logarithm)

- Fully accounts for self-gravity and inhomogeneity

- *Already been used to describe 2D galactic disks (Fouvry+ 2015), where self-gravity amplifies relaxation by x1000*

cons

it's a complete nightmare

$$\frac{\partial f_0(\mathbf{J},t)}{\partial t} = -\frac{\partial}{\partial \mathbf{J}} \cdot \mathbf{F},$$

$$\boxed{\mathbf{F}(\mathbf{J})} = \tfrac{1}{2}(2\pi)^4 \mu \sum_{\mathbf{n}\mathbf{n}'} \mathbf{n} \int \mathrm{d}^3\mathbf{J}' \left|E_{\mathbf{n}\mathbf{n}'}(\mathbf{J},\mathbf{J}',\mathbf{n}\cdot\mathbf{\Omega})\right|^2 \times \left(\mathbf{n}'\cdot\frac{\partial}{\partial \mathbf{J}'} - \mathbf{n}\cdot\frac{\partial}{\partial \mathbf{J}}\right) f_0(\mathbf{J}) f_0(\mathbf{J}')\delta(\mathbf{n}'\cdot\mathbf{\Omega}' - \mathbf{n}\cdot\mathbf{\Omega}).$$

$$\boxed{E_{\mathbf{n}\mathbf{n}'}(\mathbf{J},\mathbf{J}',\omega)} \equiv \frac{1}{\mathcal{E}} \sum_{pp'} \hat{\Phi}^{(p)}(\mathbf{n},\mathbf{J}) \epsilon^{-1}_{pp'}(\omega) [\hat{\Phi}^{(p')}(\mathbf{n}',\mathbf{J}')]^*,$$

$$\boxed{\epsilon_{pp'}(\omega)} \equiv \delta_{pp'} - M_{pp'}$$

$$\boxed{M_{pp'}} \equiv \frac{(2\pi)^3}{\mathcal{E}} \int \mathrm{d}^3\mathbf{J} \sum_{\mathbf{n}} \frac{\mathbf{n}\cdot\frac{\partial f_0}{\partial \mathbf{J}}}{\mathbf{n}\cdot\mathbf{\Omega} - \omega} [\hat{\Phi}^{(p)}(\mathbf{n},\mathbf{J})]^* \hat{\Phi}^{(p')}(\mathbf{n},\mathbf{J}).$$

$$\boxed{\widehat{\Phi}^{(p)}(\mathbf{n},\mathbf{J})} = \delta^{n_3}_{m^p}\, \mathrm{i}^{m^p - n_2}\, Y^{n_2}_{\ell^p}(\pi/2, 0)\, R^{\ell^p}_{n_2 m^p}(\beta)\, W^{\tilde{\mathbf{n}}}_{\ell^p n^p}(\widetilde{\mathbf{J}}),$$

$$\boxed{W^{\tilde{\mathbf{n}}}_{\ell n}(\widetilde{\mathbf{J}})} = \frac{1}{\pi}\int_0^\pi \mathrm{d}\theta_1 U^\ell_n(r(\theta_1)) \cos[n_1\theta_1 + n_2(\theta_2 - \psi)]$$

Applying the BL equation to globular clusters

Revisiting relaxation in globular clusters

Chris Hamilton,[1,2]★ Jean-Baptiste Fouvry,[3]† James Binney[1] and Christophe Pichon[4,5,6] *(2018, MNRAS)*

Wanted to evolve f_0 using the BL equation, and compare to Chandrasekhar's prediction.

But this is much harder than integrating the usual (homogeneous) Fokker-Planck equation!

Had to restrict ourselves to:

- Simply evaluating $F(J)$ (and hence df_0/dt) at $t=0$
- Small ℓ (i.e. only compute largest scale fluctuations)
- Small k, k' (i.e. only consider simple resonances)

Results

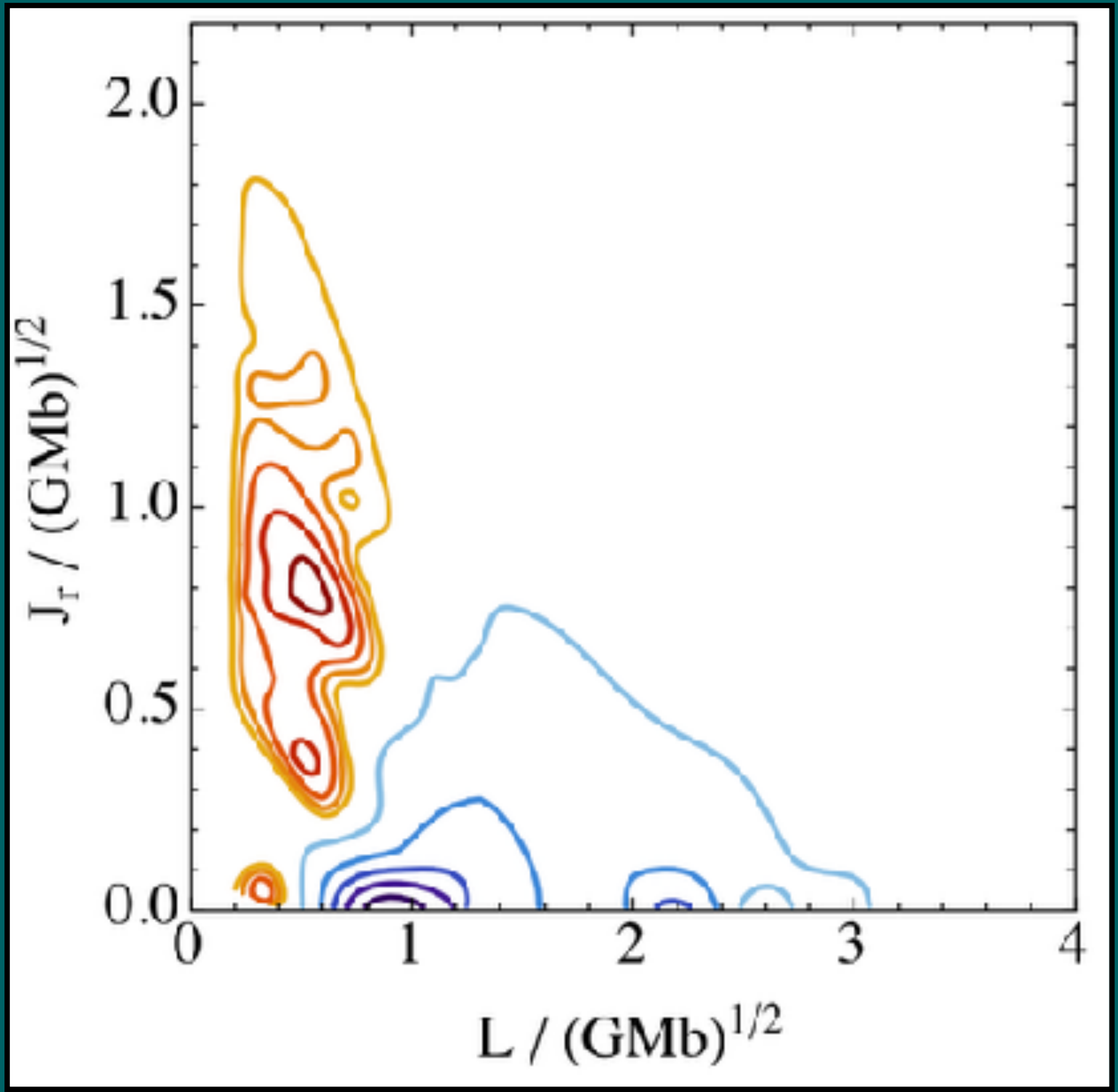
- ℓ=1 (dipole) mode of cluster is easily excited and drives significant evolution

- The flux F from this mode is amplified by self-gravity by a factor 10-100

- Its effect is to put stars on less radial orbits with more angular momentum (at roughly the same energy)

$df_0/dt > 0$

$df_0/dt < 0$



Hamilton et al. 2018

Results

- Predicted flux from BL theory is very different from Chandrasekhar theory!
- This is expected: Chandrasekhar only deals with small scales (self-gravity unimportant)
- So far BL only deals with largest scales
- But both fluxes are of comparable magnitude
- Basic conclusions of Hamilton et al. (2018) were recently affirmed by extraction of $F(J_r,L)$ from N-body simulations by Lau & Binney (2019) (arxiv: 1906.11651)

Conclusions

- System-scale resonant fluctuations amplified by self-gravity (collective effects) certainly drive evolution of galactic disks (Fouvry+ 2015).

- They likely also drive significant evolution of globular clusters.

- Our best guide to the physics of large-scale relaxation comes from the Balescu-Lenard equation.

- Chandrasekhar's 'two-body relaxation' theory is incomplete.

Revisiting relaxation in globular clusters
Hamilton, Fouvry, Binney, Pichon, (2018) *MNRAS*

Recently, Lau & Binney extracted $F(J_r, L)$ from direct N-body simulations.

Relaxation of spherical stellar systems

Jun Yan Lau[1], James Binney[1*]
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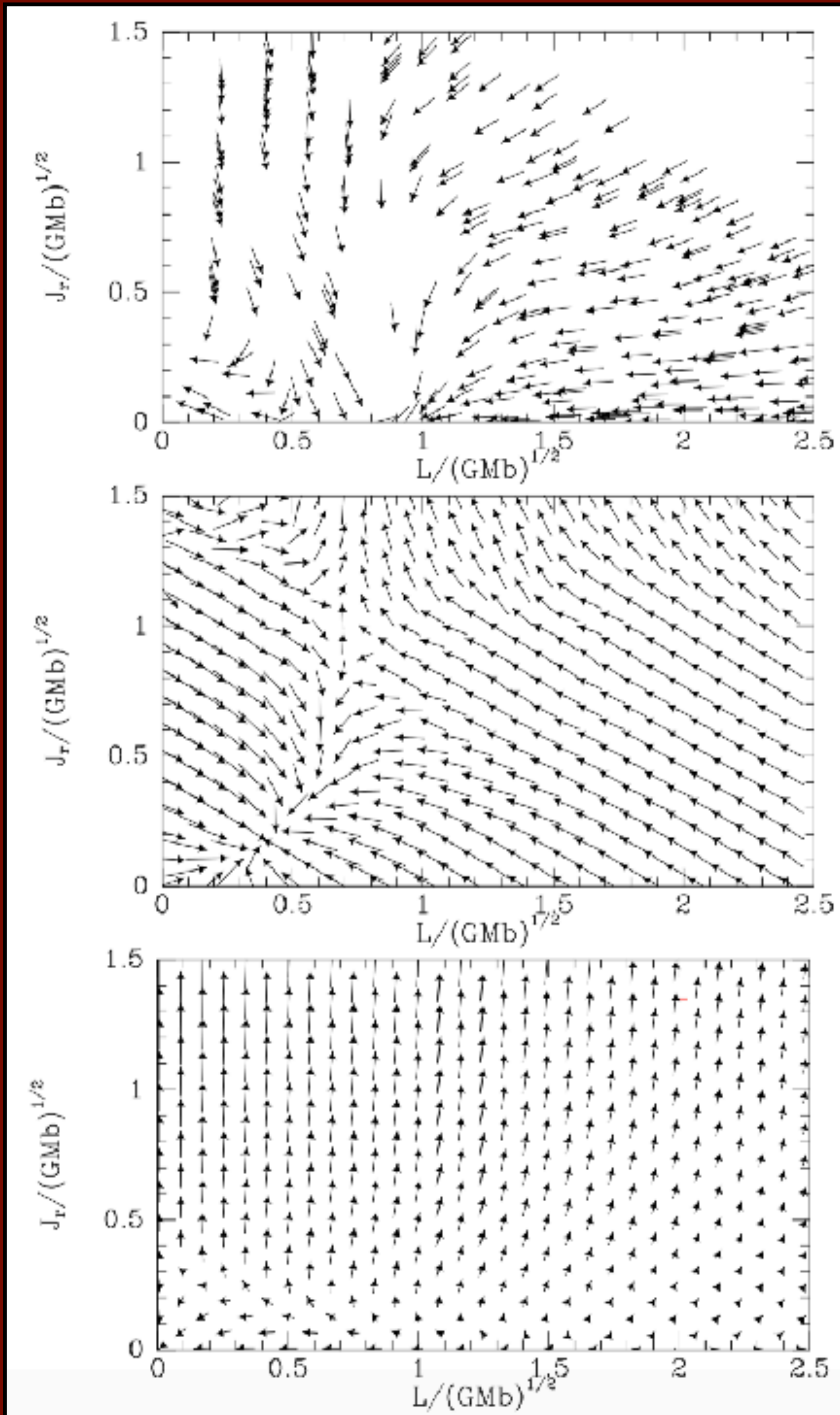
28 June 2019

(arxiv:1906.11651)

“Over three crossing times the original Poisson noise is amplified more than tenfold by self-gravity.”

“The cluster’s fundamental dipole mode is strongly excited …[and] makes a major contribution to driving diffusion of stars in energy.”

Lau & Binney (2019)



Balescu-Lenard prediction (Hamilton+ 2018)

Flux extracted from N-body simulations

Chandrasekhar's two-body relaxation theory prediction