Transport driven by trapped particle turbulence in magnetized plasmas

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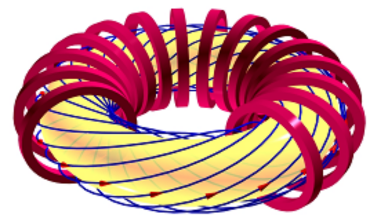
Magnetic confinement

Lawson criterion $\rightarrow nT\tau_E > 3 \times 10^{21}$ keV s m^{-3}

$\Rightarrow$ Cyclotron motion + Drifts (v_{R_c} and $v_{\nabla B}$)



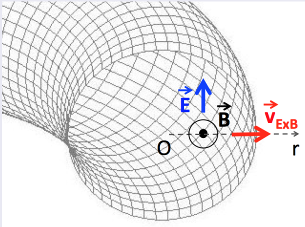
$\Rightarrow$ *a priori* : Plasma perfectly confined

$\Rightarrow$ But transport across B (collisions + **turbulence**)

Magnetic confinement

Poloidal cross section

$\Rightarrow$ Instability / Turbulence $\rightarrow \phi \neq 0 \rightarrow \vec{E} \neq \vec{0} \rightarrow \vec{v}_{E\times B} = \frac{\vec{E}\times\vec{B}}{B^2} \neq \vec{0}$



$\Rightarrow$ Turbulent $\vec{E}$: transport $\perp$ to $\vec{B}$

Impurity transport

Impurities =
All particles that are not electrons or do not contribute to the reaction of fusion

1. Argon or neon *(Tayloring the radiation profile near the plasma-facing components)*
2. Tungsten *(Transport from the wall to the core)*
3. Helium ash *(From the core to the wall)*

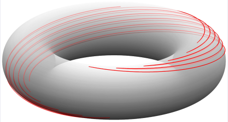
$\Rightarrow$ Impurity accumulation must be avoided

$\Rightarrow$ Impurity transport as a function of the charge number Z

Modeling trapped particles

Motion of a single trapped particle in a tokamak :

1. The fast cyclotron motion (ω_c,ρ_c),
2. The bounce (or "banana") motion (ω_b,δ_b),
3. The precession drift along the toroidal direction (ω_d,R),



with

$$\omega_d \ll \omega_b \ll \omega_c \text{ and } \rho_c \ll \delta_b \ll R.$$

Turbulence driven by trapped particles (TEM, TIM) : frequencies $\sim \omega_d$.

Gyroaveraging over the cyclotron motion + the bounce motion

$\rightarrow$ Filter out fast frequencies ω_c and ω_b and small space scales ρ_c and δ_b.

$\rightarrow$ Reduce the dimensionality from 6D to 4D :

f is the **banana center** distribution function

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$\rightarrow$ Hamiltonian mechanics $\rightarrow f = f_{\mu,E}(\psi, \alpha)$

$$\vec{\alpha} = \begin{pmatrix} \alpha_1 = \omega_c(\vec{J})t + \alpha_{10} \\ \alpha_2 = \omega_b(\vec{J})t + \alpha_{20} \\ \alpha_3 = \omega_d(\vec{J})t + \alpha_{30} \end{pmatrix} \quad \vec{J} = \begin{pmatrix} J_1 \propto \mu = \frac{m_s v_\perp^2}{2|\vec{B}|} \\ J_2 \propto E \\ J_3 \propto \psi \propto -r \end{pmatrix}$$

μ the first adiabatic invariant,
E the kinetic energy,
$\alpha_3 = \alpha = \varphi - q\theta$ ($\sim$ toroidal coordinate),
and ψ the poloidal flux ($\sim$ radial coordinate r).

Vlasov equations

$$\frac{\partial f_s}{\partial t} - \frac{\partial \mathcal{J}_{0s}\phi}{\partial \alpha}\frac{\partial f_s}{\partial \psi} + \frac{\partial \mathcal{J}_{0s}\phi}{\partial \psi}\frac{\partial f_s}{\partial \alpha} + \frac{\Omega_d E}{Z_s}\frac{\partial f_s}{\partial \alpha} = 0 \quad (1)$$

$$\frac{\partial f_e}{\partial t} - [\mathcal{J}_{0e}\phi, f_e] - \Omega_d E \frac{\partial f_e}{\partial \alpha} = 0 \quad (2)$$

with $\mathcal{J}_{0s}$ the gyro-bounce-average operator :

$$\mathcal{J}_{0s} = \left(1 - \frac{E}{T_s}\frac{\delta_{b,s}^2}{4}\partial_\psi^2\right)^{-1}\left(1 - \frac{E}{T_s}\frac{q^2\rho_{c,s}^2}{4a^2}\partial_\alpha^2\right)^{-1} \quad (3)$$

s : ion and impurity species
e : electron species

Quasi-neutrality constraint

$$\frac{2}{\sqrt{\pi}}\left[\sum_s Z_s C_s \int_0^{+\infty} J_s f_s E^{1/2} dE - \int_0^{+\infty} J_e f_e E^{1/2} dE\right] = \frac{1}{T_{i,0}}\left[C_{ad}(\phi - \epsilon_\phi \langle\phi\rangle) - C_{pol}\Delta_{i+e}(\phi)\right] \quad (4)$$

$$\Delta_{i+e} = \sum_s C_s \tau_s Z_s^2 \Delta_s + \tau\Delta_e$$

$$\Delta_s = \left(\frac{q\rho_{c0,s}}{a}\right)^2 \partial_\alpha^2 + \delta_{b0,s}^2 \partial_\psi^2$$

$$\epsilon_\phi = \frac{\tau\epsilon_{\phi_e} + \sum_s C_s \tau_s Z_s^2 \epsilon_{\phi_s}}{\tau + \sum_s C_s \tau_s Z_s^2}$$

$$\tau_s = T_0 / T_s$$

$$\boxed{C_s = n_s / n_0}$$

$$\boxed{\kappa_{n_s} = \frac{1}{n_s}\frac{\partial n_s}{\partial\psi}, \quad \kappa_{T_s} = \frac{1}{T_s}\frac{\partial T_s}{\partial\psi}}$$

$$\boxed{Z_s : \text{Charge number}}$$

Pros

- Kinetic trapped particles (ions + impurities + electrons)
- Trapped Ion Modes (TIM) and Trapped Electron Modes (TEM)
- Low numerical cost - TERESA parallelized on E, μ, and species

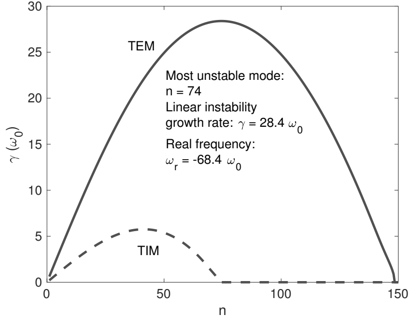
Cons

- Collisionless plasma ($\rightarrow$ ~~Neoclassical transport~~)
- Adiabatic passing particles ($\rightarrow$ ~~ITG~~, ~~ETG~~)
- $\omega \lesssim \omega_b$ for high mode numbers

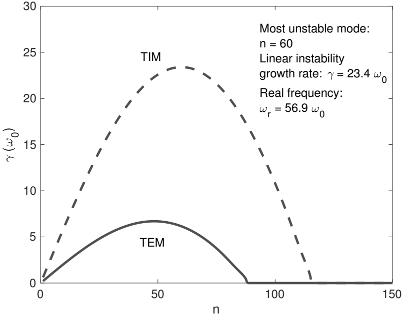
Depret *et al.*, PPCF 42, 949 (2000)
Sarazin *et al.*, PPCF 47, 1817 (2005)
Cartier-Michaud *et al.*, JPCS 561, 012003 (2014)
Drouot *et al.*, POP 22, 082302 (2015)

Linear growth rate γ as a function of the mode number n ($\omega = \omega_r + i\gamma$)

TEM $C_Z = \frac{n_Z}{n_0} = 10^{-5}$ **TIM**



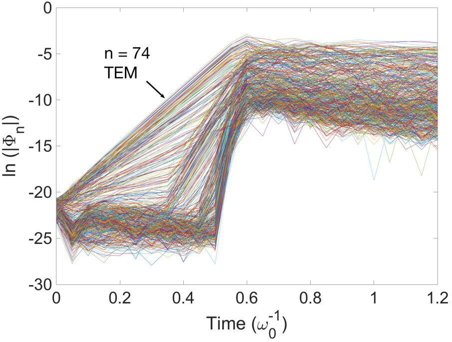
$\nabla T_e > \nabla T_i$



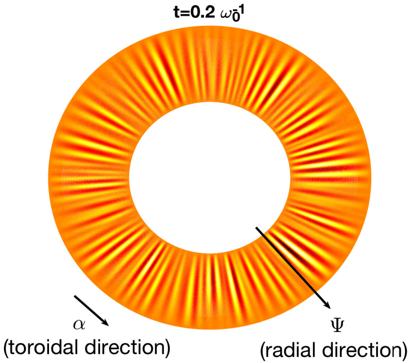
$\nabla T_i > \nabla T_e$

Example of numerical simulation : TEM (Most unstable mode $n = 74$)

Plasma potential



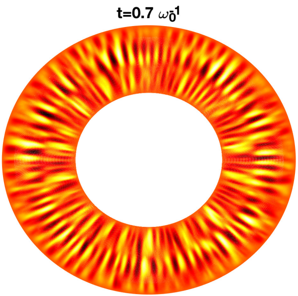
Time evolution of the α-modes



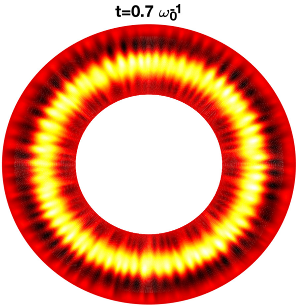
Plasma potential ϕ (ψ, α)

Example of numerical simulation : TEM (Most unstable mode $n = 74$)

Plasma potential / Zonal flows



$\phi(\psi, \alpha)$ - ϕ_{ZF}



$\phi\ (\psi, \alpha)$

with $\phi_{ZF} = < \phi >_\alpha$ (Zonal Flow)

Impurity diffusion coefficient D_Z - Method

Impurity particle flux (given by TERESA) :

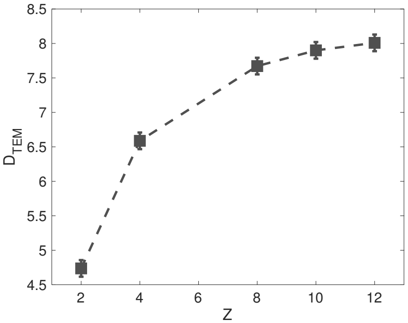
$$\Gamma_Z(t) = -\frac{2}{\sqrt{\pi}} \int_0^{2\pi} d\alpha \int_0^{\infty} f_Z \partial_\alpha (\mathcal{J}\phi) E^{1/2} dE \tag{5}$$

Impurity diffusion coefficient D_Z :

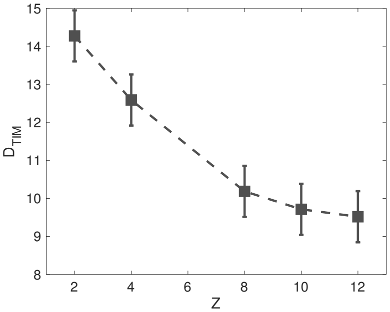
$$D_Z = -\frac{\Gamma_Z}{\partial_\psi n_Z} \tag{6}$$

Impurity diffusion coefficient D_Z

Z-dependence / $A = 20$



TEM case



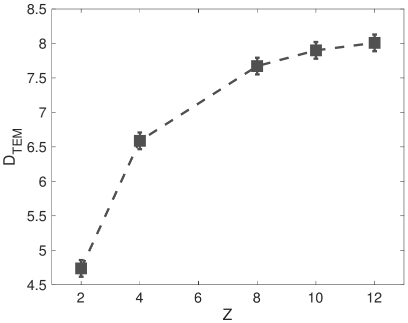
TIM case

TEM case : $Z \nearrow \Rightarrow D \nearrow$

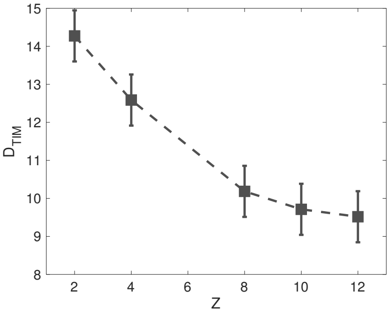
TIM case : $Z \nearrow \Rightarrow D \searrow$

Impurity diffusion coefficient D_Z

Z-dependence / $A = 20$



TEM case



TIM case

For large $Z \Rightarrow D \sim$ constant

(in agreement with [Angioni *et al.*, Nucl. Fusion, 2017])

Quasi-linear theory

$$\frac{\partial f_0}{\partial t} - \frac{\partial}{\partial \psi}\left(\sum_n |n\mathcal{J}\phi_n|^2 \kappa_n F_{eq_{\psi=0}} \frac{\gamma_n}{(n\frac{\Omega_d}{Z}E - \omega_{r,n})^2 + \gamma_n^2}\right) = 0$$

Integrating over energy $E \Rightarrow \frac{\partial n_0}{\partial t} + \frac{\partial \Gamma_\psi}{\partial \psi} = 0$

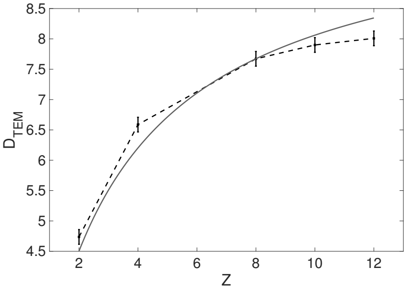
with

$$\boxed{\Gamma_\psi = -\kappa_n \int_E \sum_n |n\mathcal{J}\phi_n|^2 \frac{\gamma_n}{(\omega_{r,n} - n\frac{\Omega_d}{Z}E)^2 + \gamma_n^2} F_{eq_{\psi=0}} E^{1/2} dE} \tag{7}$$

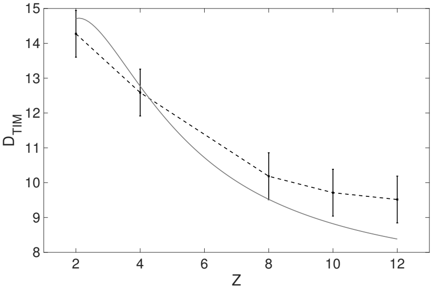
Z-dependence : QL calculations versus simulations

TEM

TIM



TEM case



TIM case

Dotted line : Numerical simulations
Solid line : QL calculations

Conclusion

- Importance of the nature of the instability that drives turbulence for the Z-dependence

  TEM case : $Z \nearrow \Rightarrow D \nearrow$ **TIM case :** $Z \nearrow \Rightarrow D \searrow$

- Impurity diffusion coefficient D is found to decrease with A (mass number) in both cases (TEM and TIM)
- QL predictions in qualitative agreement with numerical simulations
- Purely diffusive impurity transport

Conclusion

- Curvature pinch ? Thermodiffusive pinch ?

  Pinch based on turbulent thermodiffusion :

  Coppi PRL 1978 / *Angioni and Peeters PRL 2006* / *Diamond et al. Modern Plasma Physics 2010* / *Guo et al. Phys. Plasmas 2016*

$$\boxed{\Gamma_{r_s} = -D_s \nabla_r n_s + V_r n_s}$$

$$V_r \propto \left| \frac{1}{T_s} \partial_r T_s \right|$$

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THANK YOU FOR YOUR ATTENTION

E. Gravier *et al.*, Phys. Plasmas, accepted (2019)
J. Médina *et al.*, Phys. Plasmas, submitted
M. Idouakass *et al.*, Phys. Plasmas 25, 062307 (2018)
J. Médina *et al.*, Phys. Plasmas 25, 122304 (2018)
M. Lesur *et al.*, Phys. Plasmas 24, 012511 (2017)
Talk by P. Morel, Vlasovia 2019 (Tuesday)

Backup slides

Dispersion relation

$$0 = C_n - \sum_s Z_s C_s \tau_s \int_0^{+\infty} J_{m,s}^2 \frac{\kappa_{n,s} + \kappa_{T,s}(\xi_s - \frac{3}{2})}{\frac{\Omega_D}{Z_s}(\xi_s - W_s)} e^{-\xi_s} \xi_s^{1/2} d\xi_s$$
$$- \tau \int_0^{+\infty} J_{m,e}^2 \frac{\kappa_{n,e} + \kappa_{T,e}(\xi_e - \frac{3}{2})}{\Omega_D(\xi_e + W_e)} e^{-\xi_e} \xi_e^{1/2} d\xi_e \qquad (8)$$

with $$C_n = \frac{\sqrt{\pi}}{2}\left[C_{ad} + C_{pol} m^2 \left(\tau \rho_{c,e}^2 + \sum_s C_s \tau_s Z_s^2 \rho_{c,s}^2\right) + C_{pol} k^2 \left(\tau \delta_{b,e}^2 + \sum_s C_s \tau_s Z_s^2 \delta_{b,s}^2\right)\right] \qquad (9)$$

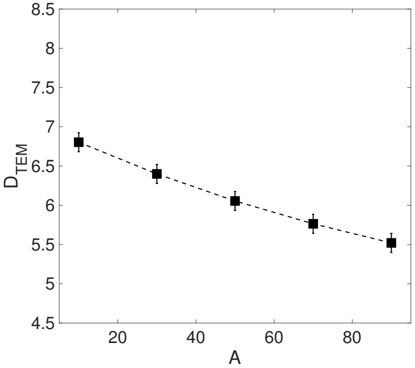
$$C_{ad} = C_{pol} \frac{1 - f_T}{f_T} \left(\tau + \sum_s C_s \tau_s Z_s^2\right) \qquad (10)$$

$$W_s = \frac{Z_s \omega}{m \Omega_D T_s} \qquad (11)$$

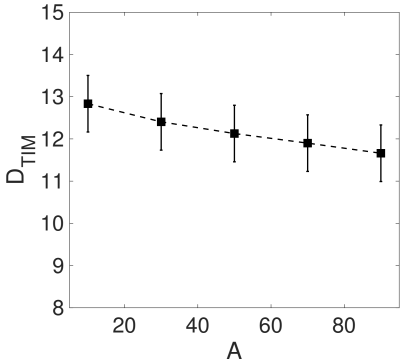
$$\xi_s = \frac{E}{T_s} \qquad (12)$$

Impurity diffusion coefficient D

A-dependence / $Z = 4$



TEM case



TIM case

Impurity diffusion coefficient D is found to decrease with A in both cases

Diffusive transport versus pinch velocity

Pinch based on turbulent thermodiffusion

Coppi PRL 1978 / *Angioni and Peeters PRL 2006* / *Diamond et al. Modern Plasma Physics 2010* / *Guo et al. Phys. Plasmas 2016*

$$\boxed{\Gamma_{r_s} = -D_s \nabla_r n_s + V_r n_s} \tag{13}$$

with $D_s = f(L_{n_e}, L_{n_s})$ and

$$V_r \propto \left| \frac{1}{T_s} \partial_r T_s \right| = L_{T_s}^{-1} \tag{14}$$

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$$V_r \propto \left| \frac{1}{T_s} \partial_r T_s \right| = L_{T_s}^{-1} \tag{16}$$

Diffusive transport : outward
Pinch velocity : competition between inward (TEM) and outward (TIM)

Diffusive transport versus pinch velocity

Pinch based on turbulent thermodiffusion

Coppi PRL 1978 / *Angioni and Peeters PRL 2006* / *Diamond et al. Modern Plasma Physics 2010* / *Guo et al. Phys. Plasmas 2016*

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Diffusive transport : outward

Pinch velocity : competition between inward (TEM) and outward (TIM)

$|V_r| \nearrow$ **when** $|\kappa_{T_s} = \frac{1}{T_s} \partial_r T_s| \nearrow$ **?**

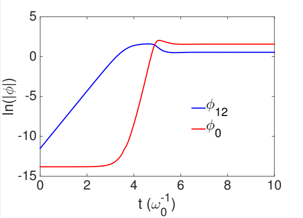
$|V_r| \searrow$ **when** $Z_s \nearrow$ **?**

Zonal flows / Predator-prey model

Itoh *et al.*, POP 13, 055502 (2006)

$$\frac{d\phi_{12}}{dt} = \gamma_{12}\phi_{12} - \eta_{12}\phi_{12}^2 - \lambda_{12}\phi_{12}\phi_0$$

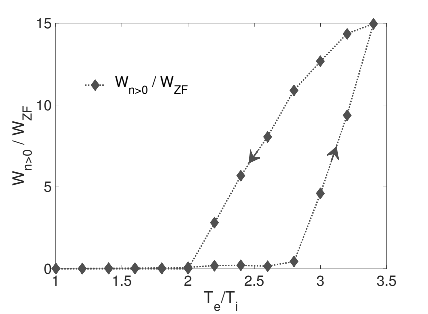
$$\frac{d\phi_0}{dt} = -\gamma_0\phi_0 - \eta_0\phi_0^2 + \lambda_0\phi_{12}\phi_0$$



Modes ϕ_{12} and $\phi_{ZF} = \phi_0$ plotted against time

Hysteresis

Gravier *et al.*, NF, 2017



$n \neq 0$ mode energy / ZF ($n = 0$) Energy
plotted against T_e/T_i

$\rho_{c,D}$	$\rho_{c,e}$	$\rho_{c,Z=2}$	$\rho_{c,Z=4}$	$\rho_{c,Z=8}$	$\rho_{c,Z=10}$	$\rho_{c,Z=12}$
6×10^{-3}	10^{-4}	9.49×10^{-3}	4.74×10^{-3}	2.38×10^{-3}	1.90×10^{-3}	1.58×10^{-3}

$\mathscr{C}_D$	$\mathscr{C}_e$	$\mathscr{C}_Z$	$\kappa_{n,D}$	$\kappa_{n,e}$	$\kappa_{n,Z}$	$\frac{T_{0_{D,Z}}}{T_{0,e}}$
0.9964	1.0	10^{-5}	0.1	0.099604	-1	1.0