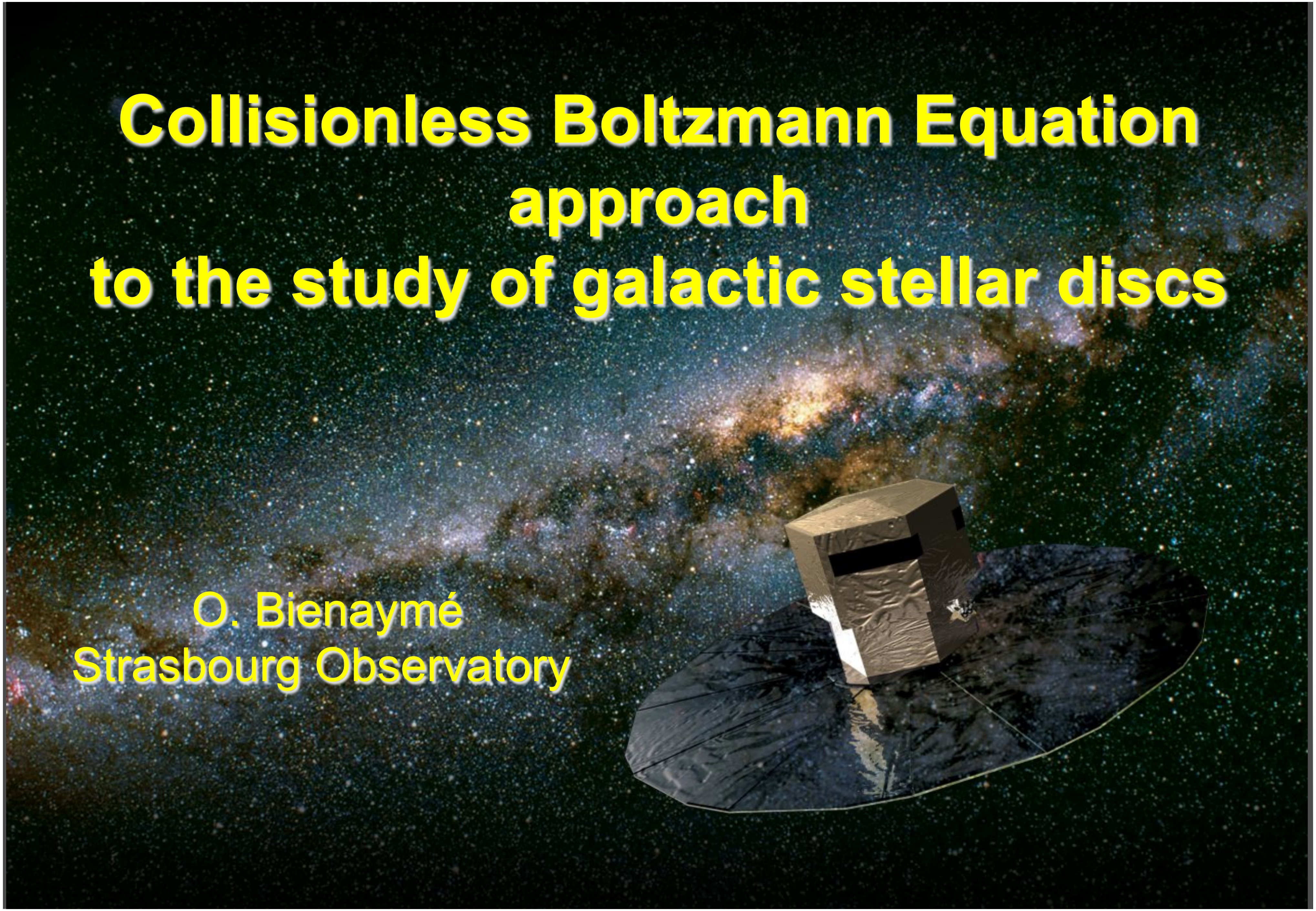

Collisionless Boltzmann Equation
approach
to the study of galactic stellar discs
O. Bienaymé
Strasbourg Observatory

« **Galactic archeology «**

Galactic formation and evolution

Chemical evolution of stars

and molecular gas clouds

Kinematics of stars and clouds

gravitational forces

and mass distribution··· DM component

Many complementary approaches for Galactic modelling,

N-body gravitational evolution

Gaia: a survey of our Galaxy

to the understanding of its formation and evolution

10^9 stars with 3D2V position and tangential velocity

a few 10^8 stars with 3D3V position and velocity (2021)

Chemical evolution of stars and molecular gas clouds

Kinematics of stars, gravitational forces

Many complementary approaches for the Galactic modelling, among them

N-body gravitational evolution

The collisionless Boltzmann equation **(i.e. N=infinity)**

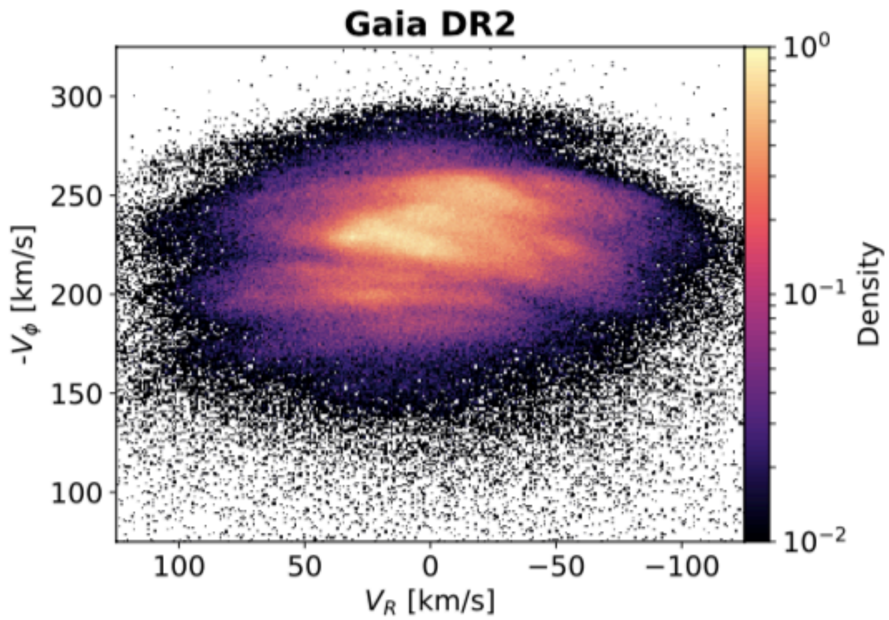
(exact DF)

(access to the modelling of very faint strutures)

Phase space section: *Gaia data*

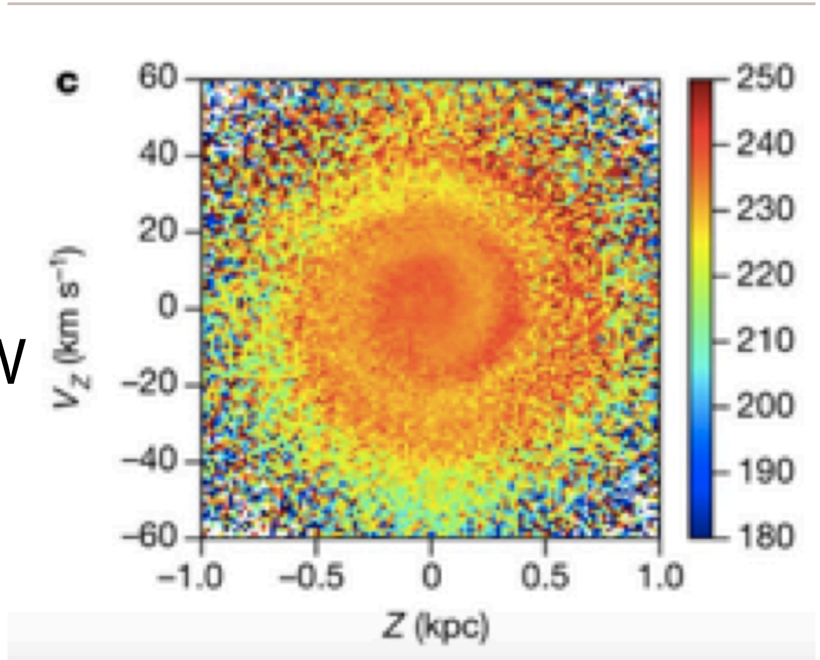
12 *Shourya Khanna et al.*

V



U

W



Z

THE EFFECT OF THE OUTER LINDBLAD RESONANCE OF THE GALACTIC BAR ON THE LOCAL STELLAR VELOCITY DISTRIBUTION

WALTER DEHNEN

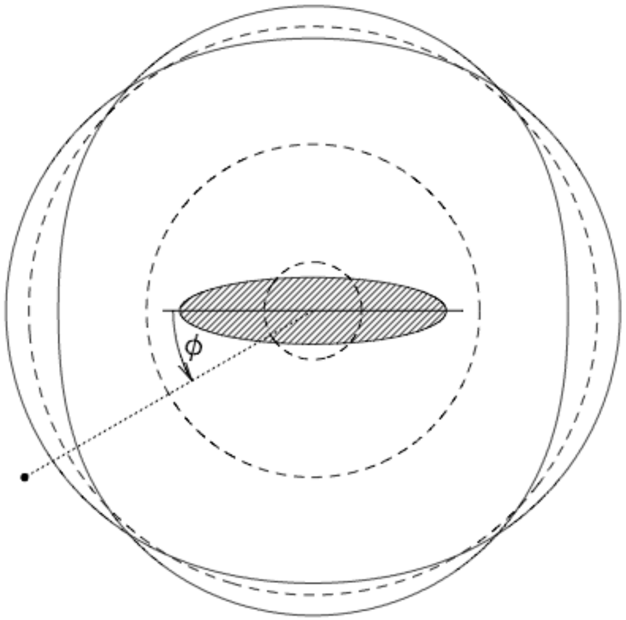


FIG. 1.—Closed orbits (*solid curves*) just inside and outside the OLR of a rotating central bar (*hatched ellipse*). The circles (*dashed curves*) depict the positions of the ILR, CR, and OLR (from inside out) for circular orbits. Note the change of the orbits' orientation at the OLR, resulting in the crossing of closed orbits at four azimuths. A possible position of the Sun is shown as filled circle. The bar angle ϕ is indicated for the case of a clockwise-rotating bar.

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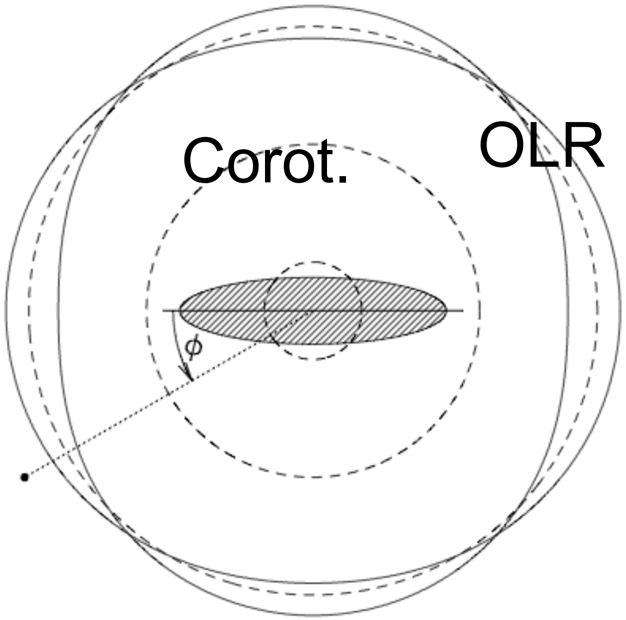

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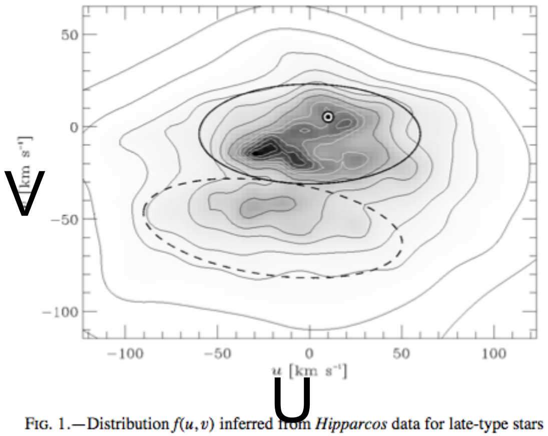

FIG. 1.—Distribution $f(u, v)$ inferred from *Hipparcos* data for late-type stars (3527 main-sequence stars with $B-V \geq 0.6$ and 2491 mainly late-type non–main-sequence stars, high-velocity stars excluded, see Dehnen 1998). Here, u and v denote the velocities toward $\ell = 0°$ and $\ell = 90°$ with respect to the LSR measured by Dehnen & Binney (1998); the circled dot indicates the solar velocity. Samples of early-type stars contribute almost exclusively to the low-velocity region (*solid ellipse*). The region of intermediate velocities (*dashed ellipse*) is mainly represented by late-type stars, ~15% of which fall into this region.

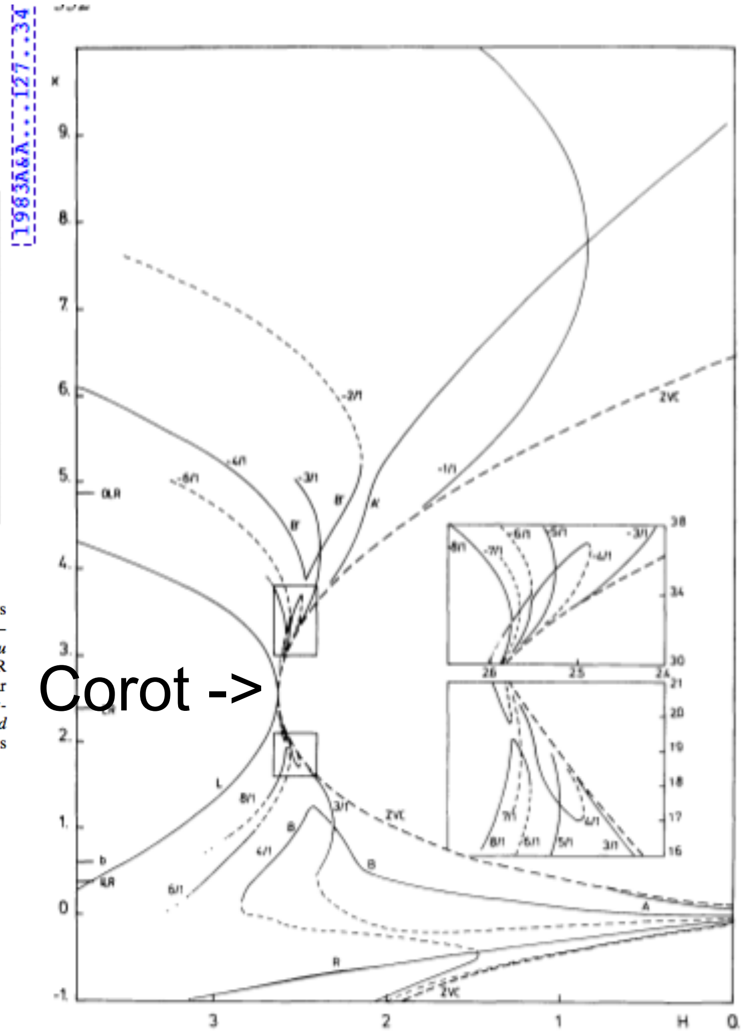

Fig. 2. Characteristics of simple periodic orbits: (H, x) diagram for the set of parameters: $\Omega_p = 0.5$, $a/r_{CR} = 1$, $b/a = 0.25$, $\varepsilon = 0.1$, $\delta = 0.8$. ZVC is the zero velocity curve (– – –) limiting the accessible region

Collisionless Boltzmann equ.

$$\frac{\partial f}{\partial t} + v \cdot \nabla_x f + F(t, x, v) \cdot \nabla_v f = 0.$$

Splitting method

$$\frac{\partial f}{\partial t} + \nabla \cdot (Af) = 0.$$

2D2V cartesian coord.

Finite difference upwind scheme

$$\bar{f}_i(t+\Delta t) = \sum_{j=-2}^{1} [\delta_{0,j+\alpha} + A_j(Q)] \bar{f}_{i+j+\alpha}(t),$$

correct at second order in Δt and Δx

where $Q = v\Delta t/\Delta x$ and:

$$A_{-2} = -\frac{Q}{4}(\sigma_v - Q),$$

$$A_{-1} = Q + \frac{Q}{4}(\sigma_v - Q),$$

$$A_0 = -Q + \frac{Q}{4}(\sigma_v - Q),$$

$$A_1 = -\frac{Q}{4}(\sigma_v - Q).$$

Eq. (24) is known as *Van Leer's scheme* [9–13]
to verify that:

Valentini et al 2005 (1D2V)
and Mangeney

Collisionless Boltzmann equ.

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Splitting method

2D2V

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Valentini et al 2005

Godunov
Lax-Wendroff
Warming
Limiter (van Leer,…)
…

<u>TESTS: 1,2,3,4D sphere, harmonic potential</u>

Numerical Simulations / Galactic disc

Initial Conditions: Shu distribution function *(for an axisymmetric disc)*

$$fd = exp[-(E-Ec(Lz))/\sigma^2]$$

V circular=1 Vel_dispersion=0.05,

grid N**4 (x,y,u,v)   N= up to 424

Models with a Rotating Bar : *(the mass of the bar is progressively increased)*

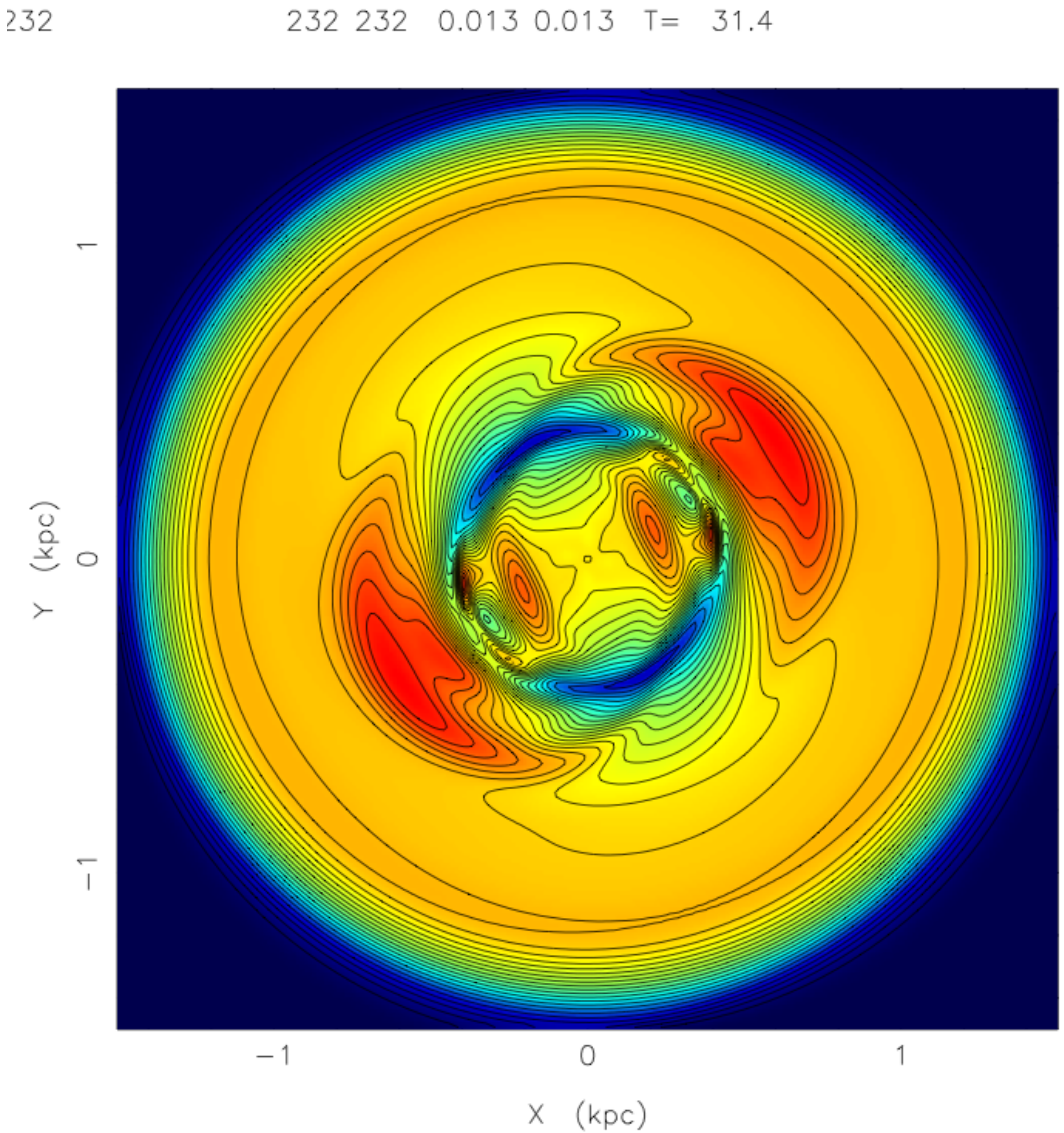
1) ½ resonance at the sun position

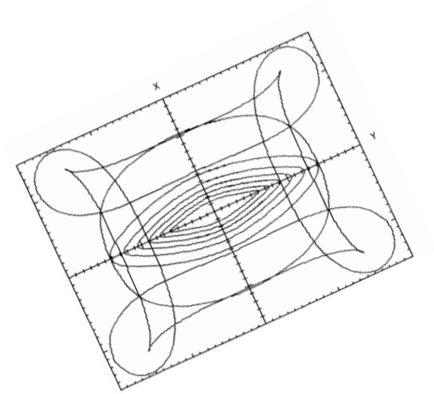
2) corotation at the sun position

Forces: Fbar / Fdisque 0.01-0.10

T final = 20 rotations *(4Gy)*

Numerical Simulations





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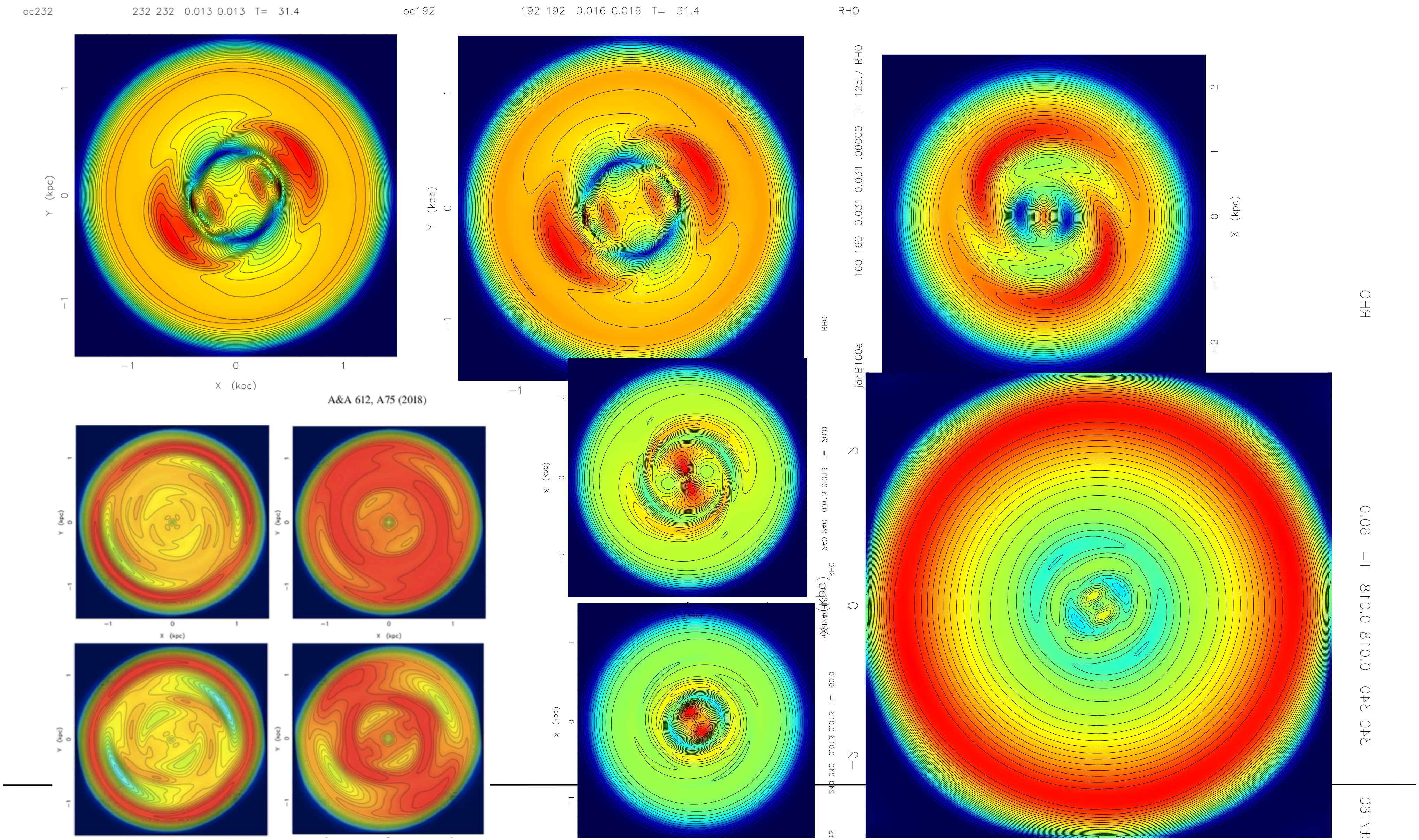
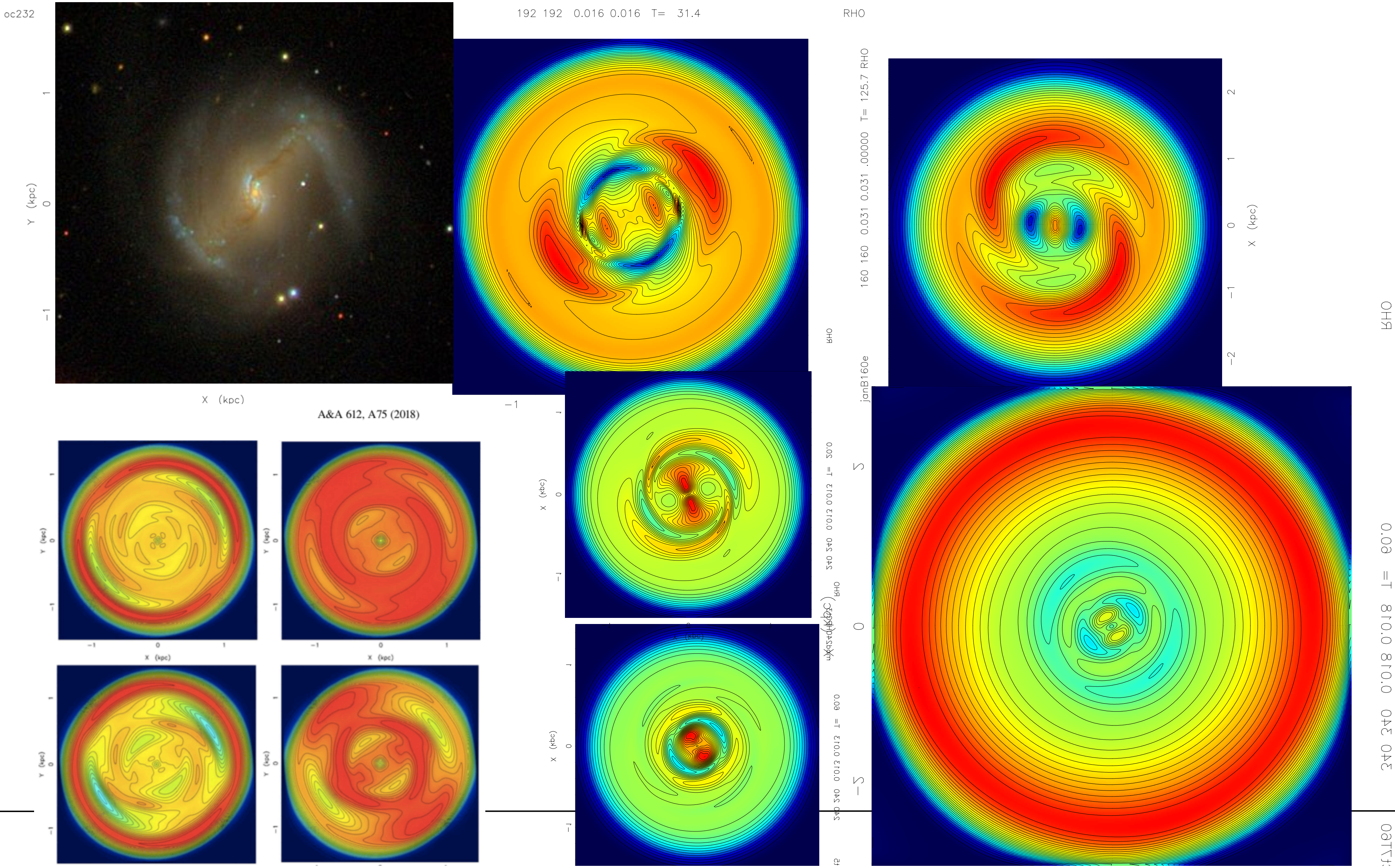
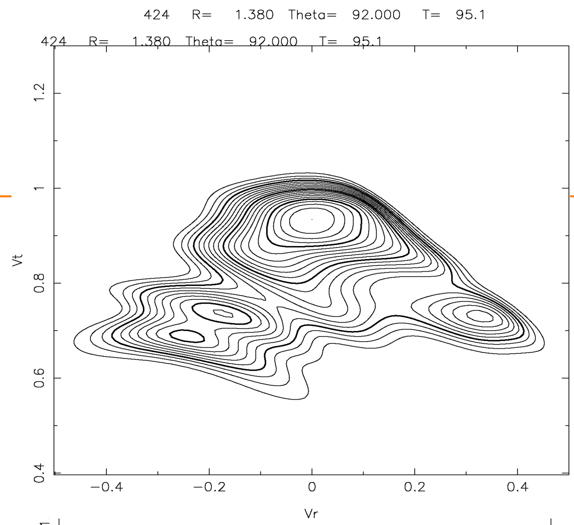


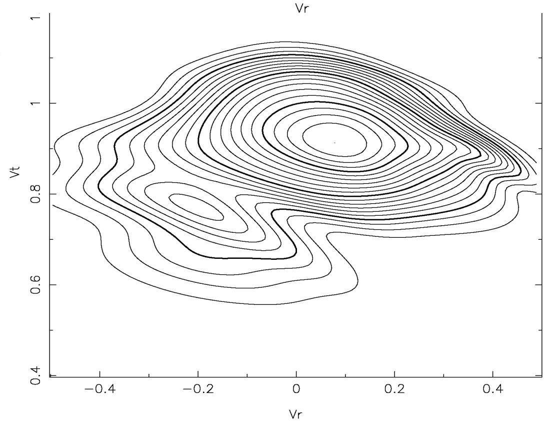
Numerical Simulations



Close to the Outer Linblad Resonance (1/2)



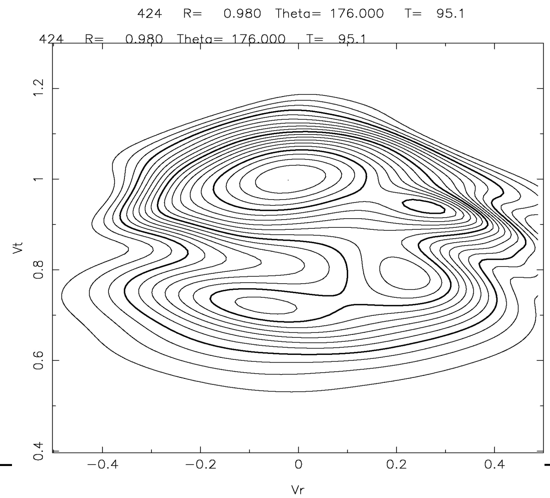
R=10.5kpc



R=8kpc

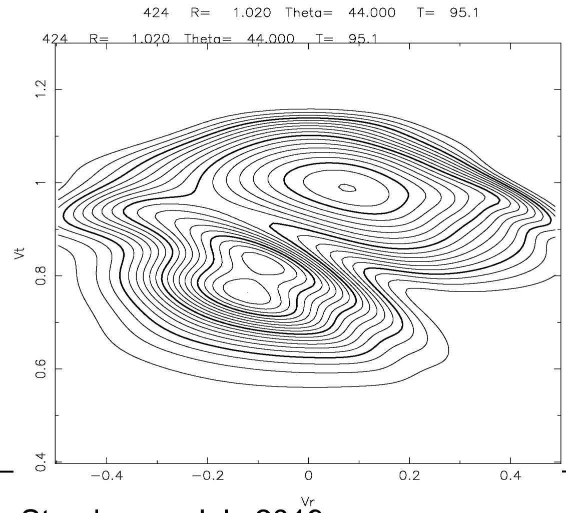
Vr

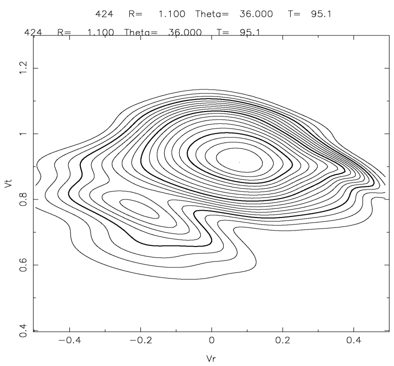
Along the bar



V_θ

Perpendicular to the bar



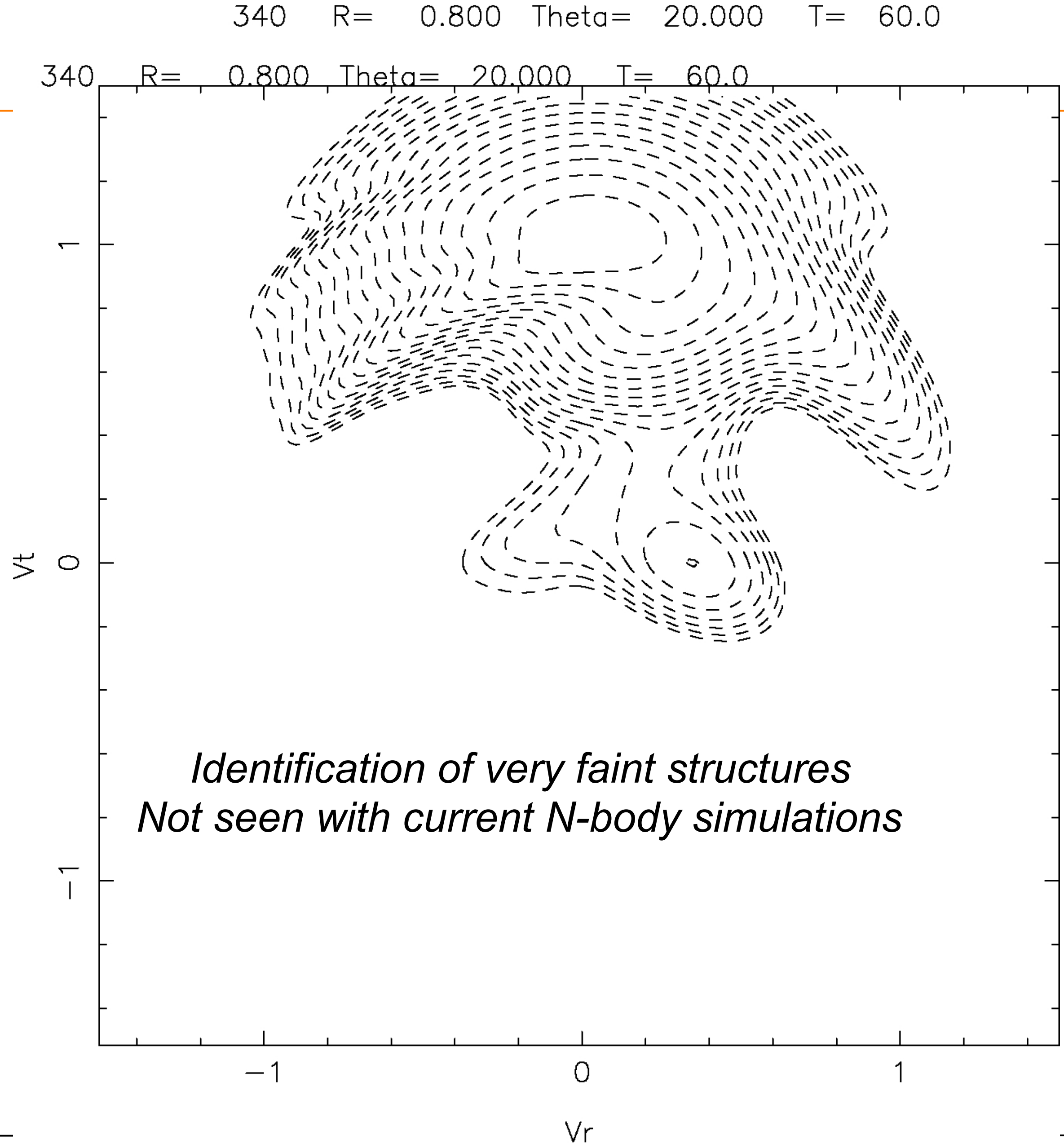


<u>Many other posibilities</u>

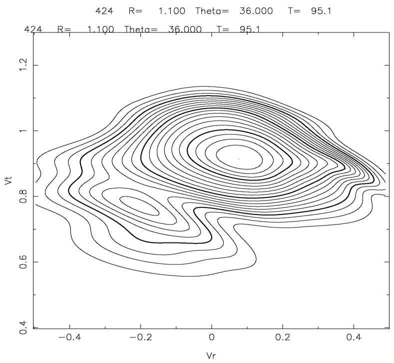
Rotating bar
Co-rotating arms
Transient arms
Accretion (ringing galaxy)
…

Different signatures according to the position within the Galaxy

Compar. to analytic works, and N-body



Identification of very faint structures
Not seen with current N-body simulations

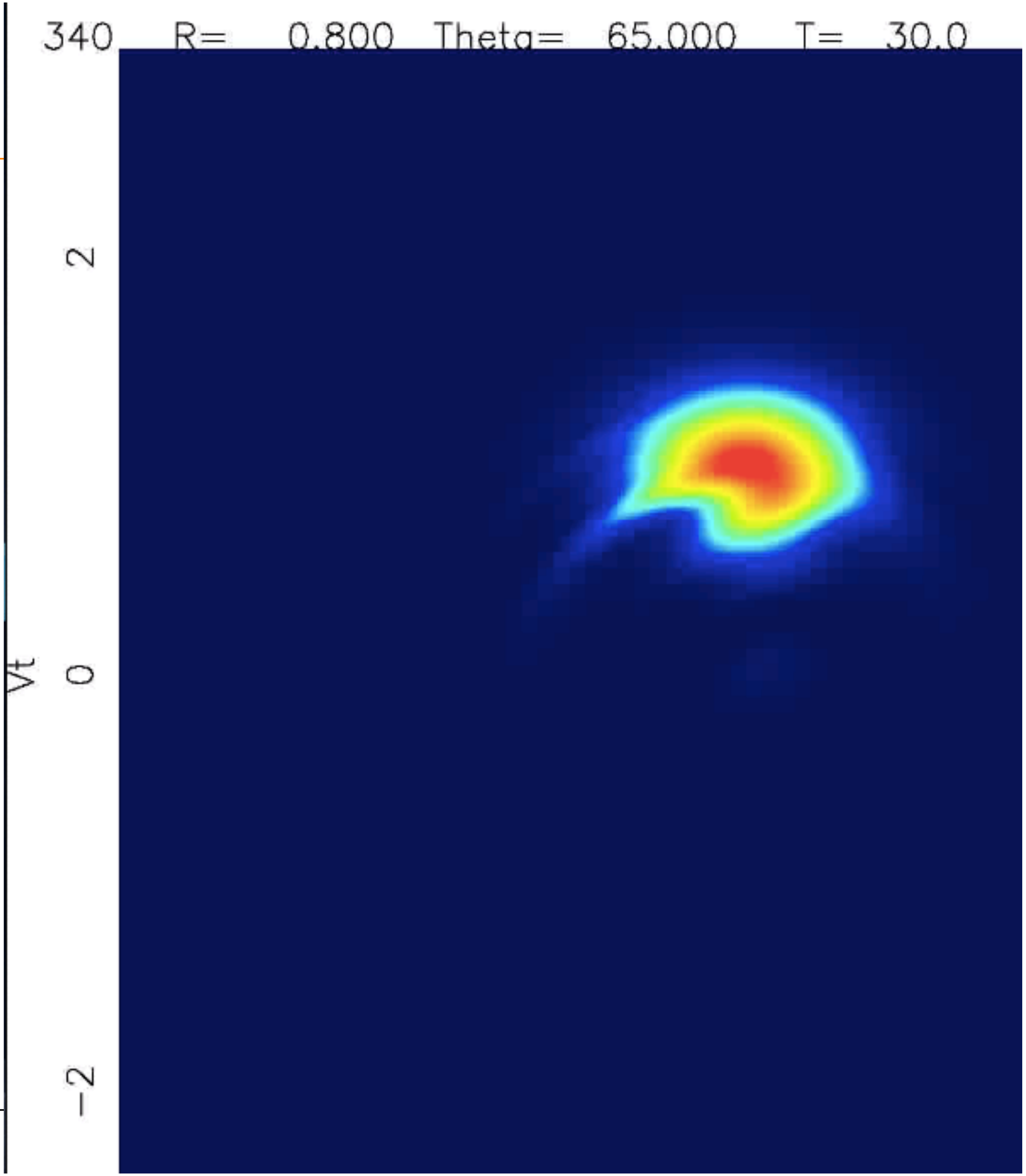


<u>Many other possibilities</u>

Rotating bar
Co-rotating (or not) arms
Transient arms
Accretion (ringing galaxy)
Accretion of DM clumps ?
….

Different signatures according to the position within the Galaxy

10^8 stars with 3D3V in 2021



Vlasov ?

Astron. Astrophys. 114, 211–212 (1982)

Research Note

Vlasov Equation?

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Received June 7, accepted June 21, 1982

Summary. Equation (1), often called today the "Vlasov equation", was already used in 1915 by Jeans, who showed that the basic set of equations for galactic dynamics is formed by (1) coupled with Poisson's equation. Vlasov's subsequent contribution was to show that, in a very similar way, the appropriate set of equations for a plasma is formed by (1) coupled with Maxwell's equations in many cases. It is suggested that the proper name of Eq. (1) should be "collisionless Boltzmann equation".

Key words: Vlasov equation – *n*-body problem – plasma physics

Three recent papers (Fujiwara, 1981; Watanabe et al., 1981; Nishida et al., 1981) use the expression "Vlasov equation" to describe the fundamental equation of collisionless stellar dynamics

$$\frac{\partial f}{\partial t}+v\frac{\partial f}{\partial r}-\frac{\partial \psi}{\partial r}\frac{\partial f}{\partial v}=0. \qquad (1)$$