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Collective Effects Triggered by Individual Effects in One-Dimensional Plasmas

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Abstract: The relationship between individual and collective effects in a two-component plasma is investigated in the case of an unstable equilibrium given by a cold two-stream distribution. The full dynamics of this system is solved using an exact *N*-body code. As the graininess parameter is large, such a cold plasma should be dominated by individual effects. Indeed, during an initial phase much longer than the plasma period, ions and electrons simply oscillate around each other forming neutral "molecules." Subsequently, however, the system switches to a regime where collective effects are important: the two-stream configuration becomes unstable and phase space structures appear. On a longer time scale, the streams are destroyed and the system evolves towards thermal equilibrium. The present results show that collective effects can emerge even in a plasma dominated by individual interactions, provided that the initial distribution is unstable.

Keywords: Strongly correlated plasmas, two-stream instability, numerical simulations

1. INTRODUCTION

A plasma is a system composed of a large number of charged particles interacting via Coulomb forces (electromagnetic effects will be neglected here).

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For such a system, interactions can be divided into *collective* (i.e., resulting from the mean field created by all other particles) and *individual* (i.e., binary collisions). The importance of collective versus individual effects is measured using the so-called graininess parameter

$$g = \frac{1}{n\lambda_D}, \tag{1}$$

defined, for a one-dimensional (1D) plasma of density n, as the inverse of the number of particles contained in a Debye length $\lambda_D = v_T/\omega_p$. Here, $v_T = \sqrt{(k_B T/m)}$ is the thermal speed and $\omega_p = (n\sigma^2/m\varepsilon_0)^{1/2}$ is the plasma frequency; m and σ are the mass and charge per unit area, respectively. Collective effects are dominant when $g \ll 1$, in which case the plasma is said to be collisionless, and its dynamics is accurately described by a Vlasov equation (coupled to Poisson's equation to compute the electrostatic mean field). The graininess parameter is also related to Γ, the ratio of total potential to kinetic energy. For a 1D plasma at thermal equilibrium, one has $\Gamma = g/2$.

For a stable equilibrium, individual effects are described, to first order in g, by the collision integral of Lenard (1960) and Balescu (1960), which was tested numerically (Rouet and Feix 1991; Ricci and Lapenta 2002) for a stable "water-bag" type equilibrium. First-order effects in the graininess parameter g were studied by Dawson (1962) in the case of a one-component plasma at thermal (Maxwellian) equilibrium, and more recently by Rouet and Feix (1998, 1996) for two other Vlasov equilibria (double water-bag and Lorentzian velocity distributions). Numerical results recover with good accuracy the theoretical predictions for the electric field fluctuation spectrum, and even for the shape of the dynamical cloud of a test particle, which is the central concept in the description of a weakly correlated plasma.

The standard Lenard–Balescu theory should not apply to an unstable equilibrium, as a crucial assumption is that the dispersion relation possesses no poles with positive imaginary part (Balescu, 1963). A generalization of the theory was attempted by Balescu (1963): it yields extremely complicated equations that are manageable only for weakly unstable plasmas, which is not the case for the instability considered here. In the present work, our aim is to perform numerical simulations that clarify the relationship between individual and collective effects in the case of a one-dimensional, two-component plasma (ions and electrons will have the same mass and charge in absolute value) for an unstable two-stream equilibrium. Finally, for a theoretical study of the equilibrium statistical mechanics of a 1D plasma, we refer the reader to the works of Kunz (1974) and Choquard (1980).

Our strategy is to study the plasma dynamics as a full N-body problem, without making any assumptions on the dominance of individual or collective effects. In order to do so, we shall use a numerical code that solves the exact N-particle dynamics (Rouet and Feix 1991) without any numerical approximations other than the inevitable truncation errors resulting from the finite number of bits used to represent real numbers. This powerful tool enables us to follow the plasma evolution over very long time scales.

The two-stream instability is a well-known process occurring in collisionless (Vlasov) plasmas and has received a lot of attention in the last few decades (see, for instance, Knorr 1968; Freidberg and Armstrong 1968; Biskamp and Chodura 1973; Goldman 2000). More recently, a quantum version of the two-stream instability was also studied both analytically and numerically (Haas 2000). The initial condition is given by a two-stream distribution, with half of the particles (both electrons and ions) traveling to the right with velocity a, and the other half traveling to the left with velocity $-a$. The instability rate can be computed from the linearized Vlasov-Poisson equations, and numerical results obtained from Vlasov simulations are in agreement with the theoretical estimate (Ghizzo et al. 1988). For completely cold streams ($v_T = 0$), only perturbations for which $k\lambda_* < 1$ are unstable, and their growth rate is:

$$\gamma_k = \omega_p \sqrt{-k^2\lambda_*^2 - \frac{1}{2} + \frac{1}{2}\sqrt{1 + 8k^2\lambda_*^2}} \tag{2}$$

where $\lambda_* = a/\omega_p$. The maximum growth rate $\gamma_{\max} \simeq 0.35\omega_p$ is obtained for $k\lambda_* \simeq 0.6$. Although the Vlasov picture should not, in principle, be appropriate for the case of zero-temperature streams, we shall see that Equation (2) *does* reproduce well the observed growth rate, even for plasmas that are relatively far from the Vlasov limit ($g \simeq 1$).

In this article, we shall study the full N-body problem in a regime corresponding to a graininess parameter $g > 1$. In order to gain some insight, let us first consider the case of a single cold stream with zero velocity. If the positions of the ions and electrons are initially distributed at random, the total potential energy of the system will generally be large (because clumps of positive or negative charge can be formed). As soon as the system is let to evolve, this potential energy will turn into kinetic energy thus destroying the stream structure. The potential energy can be reduced by matching ions and electrons one by one to form ion-electron pairs. If the ion and the electron in each pair are close enough, the system is in fact constituted of neutral "molecules." Further, in 1D, the electric field outside each molecule will be equal to zero, so that no interactions between the molecules should occur. As the one-stream distribution is stable from the viewpoint of Vlasov theory, the plasma will remain indefinitely in this configuration and no collective effects are expected to emerge. In order to have interaction between two neighboring molecules, the size of either of them should be large enough, so that one of its particles can penetrate into the other molecule. By progressively increasing the size of the molecule, Bonomi (1978) studied numerically the propagation of a wave into a one-component plasma at zero temperature.

In the two-stream case, we prepare the system in an similar way, by matching ions and electrons one by one on each stream. Therefore, each molecule has a velocity equal to either a or $-a$. For the same reasons as given in the previous paragraph, molecules belonging to the same stream should not directly interact with each other. However, two molecules from different streams can indeed interact when they cross each other. In this case, it is possible that, after a number of collisions, the molecules are somehow destroyed, leading to a change in physical behavior. This interaction mechanism is naturally of an *individual* (not collective) nature. Therefore, it is not obvious that it will lead to the excitation of the same two-stream instability as observed in collisionless systems. This question will be investigated in the rest of this article.

2. NUMERICAL METHOD

Numerical simulations have been performed to follow the evolution of a 1D two-component plasma: the system is composed of $N/2$ ions and $N/2$ electrons of equal mass m and charge σ (in absolute value) by unit area, and periodic boundary conditions are assumed. Note that these 1D "particles" correspond (in 3D) infinite parallel plane sheets normal to the x axis. The restriction to 1D systems is justified for our purposes, as the relevant physical ingredient are still present in the 1D model. Further, this assumption allows us to follow the exact trajectories of the N particles without any approximations (Rouet and Feix 1991). This is because: (a) a relation of order between the positions of the particles exists in a 1D geometry, and (b) the electric field created by the particles is piecewise constant. The motion of a particle is thus uniformly accelerated until it reaches its neighbor on either side. When such an event occurs, as the 1D electric field has no divergence, the particles are allowed to cross each other. After the crossing, the field is changed locally and the particles will experience a new constant acceleration until the next crossing event. The code follows the trajectories of the N particles without any numerical approximations, except the round-off errors due to the finite number of bits used to code a real number on the computer (Rouet and Feix 1991). We stress that this is *not* a particle-in-cell (PIC) simulation: PIC codes solve the Vlasov mean-field dynamics by integrating the orbits of a large number of discrete particles, whereas our code solves the exact N-body dynamics, including both mean-field and individual effects.

The initial condition is prepared as follows: the ion-electron pairs (molecules) are spaced regularly on each stream and all pairs have a velocity close to either a or $-a$. The ion and the electron in each pair are located at a distance $\pm\delta$ from the center of mass of the molecule, with δ chosen at random with equiprobability in the interval $[0, \Delta]$. Here, Δ represents the average size of a molecule and is also proportional to its total energy in the reference frame of the stream. The field generated by a pair of planar particles separated by a distance 2δ is spatially uniform and equal to σ/ε_0. The corresponding potential energy is $E_{\rm pot} = \sigma^2\delta/\varepsilon_0$. All molecules are given the same total energy in the reference frame of the stream: $E_{\rm tot} = mv^2 + \sigma^2\delta/\varepsilon_0 = \sigma^2\Delta/\varepsilon_0$ (the latter equality is obtained by noting that $v = 0$ when $\delta = \Delta$). The particle velocity is thus given by the expression

$$v^2 = \frac{\sigma^2}{m\varepsilon_0}(\Delta - \delta) = \omega_p^2\frac{\Delta - \delta}{n}. \tag{3}$$

With the above prescriptions, all molecules will initially oscillate with the same amplitude (proportional to Δ) and random phases.

As an approximate measure of the thermal speed of the streams, we take the maximum velocity that can be obtained from Equation (3): $v_T^2 = \omega_p^2\Delta/n$. This leads to the following expressions for the Debye length $\lambda_D = \sqrt{(\Delta/n)}$ and, using Equation (1), for the graininess parameter

$$g = \frac{1}{\sqrt{n\Delta}}. \tag{4}$$

The latter expression implies that, if the size of a molecule 2Δ is smaller than the average distance between two molecules $(2n^{-1})$, then $g > 1$ and the plasma is dominated by individual effects. For a single cold stream, this means that the molecules cannot, in this case, cross each other, and thus they preserve their identity over indefinitely long times. For a two-stream distribution, molecules from different streams are allowed to collide, so that the intermolecular distance can in principle change in time. In order to switch to a collisionless regime $(g < 1)$, the molecules size must become larger than the intermolecular distance: but this means, in practice, that the molecules are destroyed. It is clear, therefore, that the existence of stable ion-electron pairs (molecules) is linked to the observed plasma regime (collective or individual). The details of this link will become apparent in the forthcoming simulation results.

In the simulations, time is normalized to ω_p^{-1} and lengths are normalized to the inverse density n^{-1}, which in practice amounts to assuming $\omega_p = n = 1$. For the following numerical results, we have taken $N = 8000$ particles ($N/2$ ions and $N/2$ electrons), $a = 500/2\pi \simeq 80\,\omega_p/n$, $L = N/n = 8000$ and $n\Delta = 0.08$. This yields $\lambda_* \simeq 80\,n^{-1}$, $v_T \simeq 0.28\,\omega_p/n$, and $g \simeq 3.54$. As $g > 1$, we are are in a regime where individual effects can play a significant role. The fundamental Fourier mode of the system has wave number $k_0\lambda_* = 0.0625$, so that a large number of modes are unstable (all those with $0 < k\lambda_* < 1$).

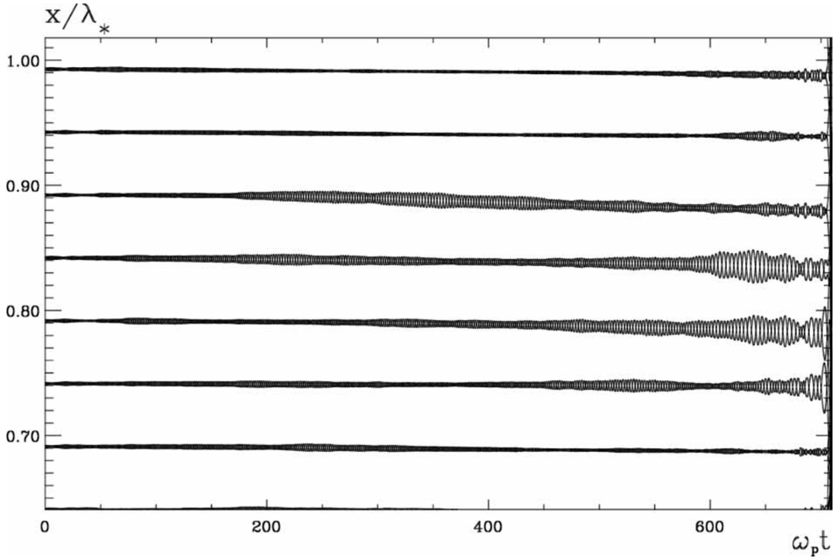

Figure 1. Trajectories $x(t)$ (normalized to λ_*) of seven ion-electron pairs in the streams reference frame, for a system with $N = 8000$ ion and electrons, $a \simeq 80$, $L = 8000$ and $n\Delta = 0.08$.

3. NUMERICAL RESULTS

Figure 1 shows the trajectories of seven pairs of particles in the reference frame of the stream they belong to. Initially, the ion and the electron of each molecule simply oscillate around each other with an amplitude Δ (however, some slight changes in the oscillation amplitude and period are observed during this phase). Although each molecule of a stream interacts (briefly, but frequently) with all the molecules of the other stream, this oscillating regime is very robust and persists on a long time scale (until $\omega_p t \simeq 700$). During this phase, the molecules preserve their identity and the plasma remains locally neutral, so that individual effects clearly dominate the dynamics. This "individual" regime suddenly breaks down around $\omega_p t = 700$, when the amplitude of the oscillations increases rapidly leading to the destruction of the pairs

Figure 2 shows the time-evolution of the system in the phase space. The two streams remain unperturbed up to $\omega_p t \simeq 680$, when the system enters a violently unstable regime. This event corresponds to the destruction of neutral molecules observed in Figure 1. After this time, the streams are quickly destroyed and coherent phase-space structures appear briefly: these are reminiscent of the vortex structures observed in Vlasov simulations (Ghizzo et al. 1988). Subsequently, such structures are damped away by collisional diffusion in phase-space, which drives the system towards thermal

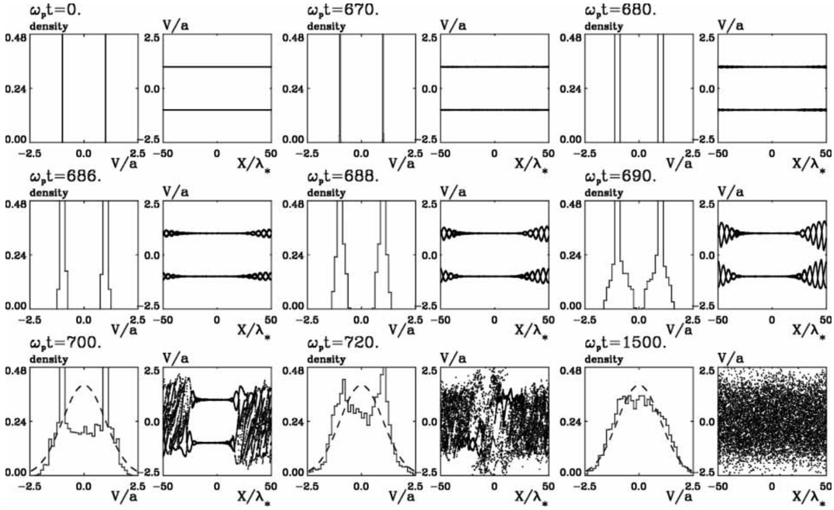

Figure 2. Phase-space evolution for the same run as in Figure 1. For each snapshot, the left frame shows the particle density in velocity space (i.e., integrated over x), the dashed line representing a Maxwellian with thermal speed equal to the stream velocities a; the right frame shows the particles in the phase-space (x, v). In this figure, lengths are normalized to λ_* and velocities are normalized to a.

equilibrium, as shown in the last picture of Figure 2 ($\omega_p t = 1500$) (Lenard 1960; Balescu 1960; Rouet and Feix 1991; Ricci and Lapenta 2002). After thermalization, the plasma remains essentially Maxwellian with a thermal speed close to the original stream velocity a. At this stage, as the streams have been completely destroyed, the original graininess parameter g is no more relevant to describe the collisionality of the plasma. One should instead use

$$g_* = \frac{1}{n\lambda_*}, \tag{5}$$

obtained by replacing the streams thermal velocity v_T with a, and therefore the original Debye length λ_D with λ_*. Such modified graininess parameter takes the value $g_* = 0.0125$, so that the plasma is now mainly collisionless, although collisional effects still persist over long time-scales.

According to the Lenard–Balescu theory (Lenard 1960; Balescu 1960; Rouet and Feix 1991; Ricci and Lapenta 2002), the thermalization time $\tau_{\rm th}$ is proportional to the inverse of the square of the graininess parameter, as was verified numerically in Rouet and Feix (1991) and Ricci and Lapenta (2002). However, the Lenard–Balescu theory is only valid for stable equilibria and in the limit of small g (whereas here $g = 3.54$). Since the initial two-stream distribution is quickly destroyed after the occurrence of the instability, it seems more appropriate to use g_* as the relevant graininess parameter. With this choice, one obtains: $\omega_p \tau_{\rm th} = g_*^{-2} = 6400$, which is in better agreement with the results of Figure 2. However, a more detailed analysis should be performed in order to check this issue properly.

In Figure 3 we show the ratio Γ of the potential energy to the kinetic energy of the system. As expected, Γ approaches $g_*/2$ at the end of the simulation, indicating that g_* is indeed the relevant graininess parameter during the thermalization phase. The small value of Γ at the beginning of the run is due to the large kinetic energy contained in the streams velocity, which is equal to $a^2/2 = 3200$ per particle. Neglecting the thermal energy of the streams (which is small compared to a^2), and estimating the potential energy per particle as $E_{pot} = \sigma^2 \Delta/(4\varepsilon_0) = 0.02$, one obtains $\Gamma(t=0) = 0.02/3200 \simeq 6 \times 10^{-6}$, in agreement with Figure 3.

We now consider the destruction of the "individual regime" occurring around $\omega_p t \simeq 700$. As we had anticipated, this effect is linked to the destruction of the neutral molecules. From Figure 1, it appears that the size of a molecule increases shortly before the onset of the instability, probably because of collisions with molecules from the other stream. According to Equation (4), the graininess parameters should decrease with increasing molecular size. At some point, g must become small enough for collective effects to take over and trigger the two-stream instability. A typical signature of collective effects is the presence of oscillations at the plasma frequency, at least for long wavelengths (the thermal correction $ka = 0.0625$ is indeed negligible). Figure 4 shows the squared amplitude of the space and time Fourier transform of the particle density $\rho(k_0, \omega)$, where

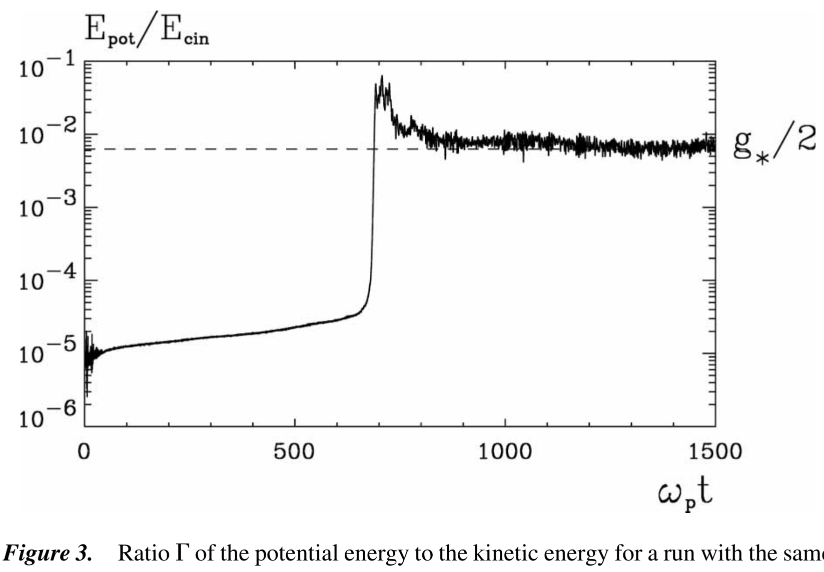

Figure 3. Ratio Γ of the potential energy to the kinetic energy for a run with the same parameters as Figure 1. The dashed horizontal line represents the value $g_*/2$.

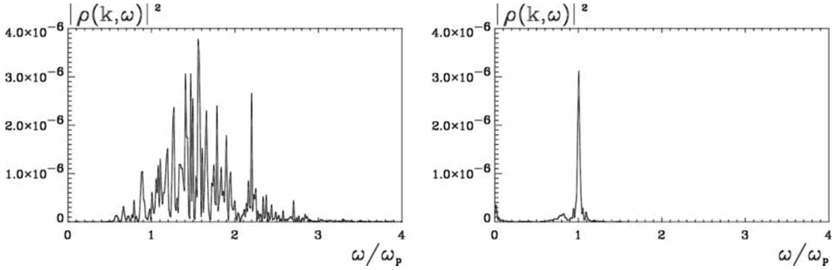

Figure 4. Square of the density fluctuation spectrum $|\rho(k_0,\omega)|^2$ for the fundamental mode k_0 for the same run as Figure 1. Left frame: fluctuation spectrum before the instability ($100 < \omega_p t < 600$); right frame: fluctuation spectrum after the instability ($600 < \omega_p t < 1100$).

$k_0 = 2\pi/L$ is the smallest wave number accessible to the simulated system (fundamental mode). The left frame shows $|\rho(k_0,\omega)|^2$ in the individual regime ($100 < \omega_p t < 600$). It displays a broad spectrum corresponding to the different periods of oscillation of the molecules, but no excitation for $\omega = \omega_p$ is observed. The right frame of Figure 4 shows $|\rho(k_0,\omega)|^2$ after the excitation of the instability ($600 < \omega_p t < 1100$). A single peak at $\omega \simeq \omega_p$ is now clearly visible, indicating that collective effects are indeed playing a major role. The previous results demonstrate that a collective phenomenon (the two-stream instability) can be triggered by individual effects (which dominated before the onset of the instability).

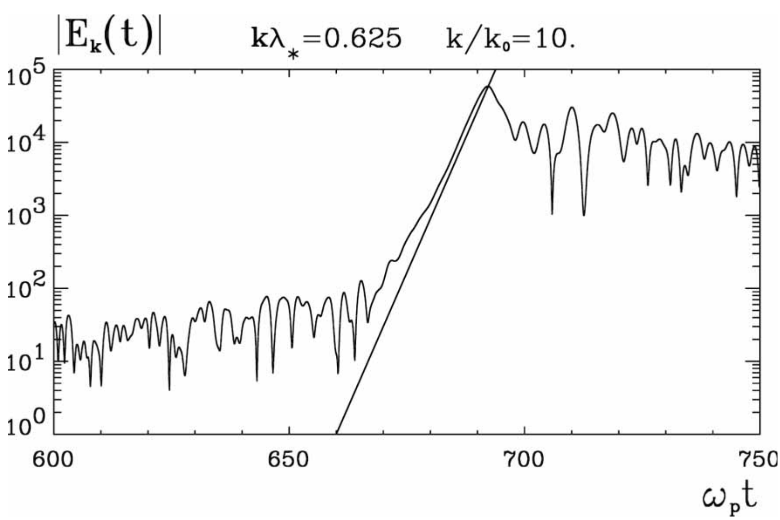

Figure 5. Time evolution of the Fourier mode with $k = 10k_0$ of the electric field. The straight line represents the growth rate given by Vlasov theory, $\gamma = 0.353\omega_p$.

A more precise measure of the impact of collective effects is given by the growth rate of the instability. Figure 5 shows the time-evolution of the most unstable Fourier mode of the electric field $E_k(t)$ on a semi-log graph. It is given for $k = 10k_0$, i.e., $k\lambda_* = 0.625$ (the maximum growth rate is attained for $k\lambda_* \simeq 0.61$, but wave numbers are discrete in a periodic system). The growth rate predicted by Vlasov theory (given in Equation (2)) is equal to $\gamma = 0.353\omega_p$ and is represented by the straight line on the figure: it fits well the computational result during the instability phase. It is therefore clear that the effect observed in Figure 5 is indeed the standard Vlasov two-stream instability.

The same kind of behavior (neutral individual regime followed by a violent Vlasov two-stream instability, and eventually thermalization) has been observed in other simulations performed with different values of a, L, and Δ.

4. CONCLUSION

The full N-body dynamics has been solved for a 1D, two-component plasma in the case of a cold two-stream initial condition. We have found that, although individual effects are not strong enough to induce macroscopic (large-scale) phenomena, they are able to trigger collective effects. Indeed, after a transient period during which the ion-electron pairs oscillate, a collective regime appears suddenly. The signature of collective effects is the appearance of long wavelength oscillations at the plasma frequency. Moreover, the growth rate of the instability is in agreement with the one given by the Vlasov theory. After the onset of the instability, coherent structures appear in the phase space, as was previously observed in Vlasov numerical simulations (Ghizzo et al. 1998). Here, these structures are quickly destroyed because of individual effects which, on a longer time-scale, drive the system towards thermal equilibrium.

In summary, the above results point out that standard arguments based on the plasma temperature and/or the graininess parameter should be taken with care when dealing with unstable equilibria such as the two-stream distribution considered here. Indeed, even when the plasma is strongly coupled ($g > 1$), the collisionless, two-stream instability may still be excited by small individual interactions. This instability, in turn, triggers the growth of large collective modes, which are eventually damped away by collisional phase-space diffusion, leading to thermal equilibrium. Thus, the plasma has gone through a sequence of three different phases: (i) the initial “individual” regime (oscillating neutral molecules); (ii) a collective nonneutral regime characterized by the instability and oscillations at frequency ω_p; and, finally, (iii) relaxation towards neutral thermal equilibrium on a longer time-scale: in this final stage, both collective and individual effects coexist, as demonstrated by the persistence of oscillations at the plasma frequency.

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