Reply to "Comment on 'Harvesting information to control nonequilibrium states of active matter' "

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We stress that the limitations on one of the results of our paper [R. Goerlich *et al.*, Phys. Rev. E **106**, 054617 (2022)], which are mentioned in the preceding Comment [A. Bérut, preceding Comment, Phys. Rev. E **107**, 056601 (2023)], were actually already acknowledged and discussed in the original publication. Although the observed relationship between the released heat and the spectral entropy of the correlated noise is not universal (but limited to one-parameter Lorentzian spectra), the existence of such a clear relationship is a solid experimental finding. It not only gives a convincing explanation for the surprising thermodynamics observed in the transitions between nonequilibrium steady states, but also provides new tools for the analysis of nontrivial baths. In addition, by using different measures of the correlated noise information content, it may be possible to generalize these results to non-Lorentzian spectra.

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In our recent paper [1], we studied the possibility of controlling the nonequilibrium states of an overdamped stochastic system (a dielectric microsphere) through a correlated noise signal generated optically with a laser beam. One of our main results was that, by changing the correlation time of the noise, the system can be driven from one nonequilibrium state to another, thereby releasing some amount of heat without any energy expenditure (although, of course, the first principle of thermodynamics is not violated, as explained in our paper). To solve this "paradox," we hypothesized that the system extracts information from the correlated noise and converts it into heat in a Landauer-like fashion. This hypothesis was confirmed experimentally by showing that the noise information (quantified by the spectral entropy of the noise spectrum) is proportional to the generated heat (Fig. 4 of [1]). Such proportionality relation was verified experimentally for a wide range of correlation times and several values of the noise amplitude.

The author of the preceding Comment [2] expresses some concerns on this aspect of our work, namely, the validity of the experimental relationship between information and heat. In particular, he raises two points: (i) that the relation of proportionality between spectral entropy and heat is not universal and (ii) that, even in the context of our work, such a relation is not rigorously verified.

Here is our response to these considerations.

The heat-entropy relation in our work was verified for the specific class of Lorentzian spectra, which depend on a single parameter, namely, the noise correlation time. For this class of spectra (by far the most important one for active matter, which was the topic of our paper), the relation is very well confirmed by the experiments. The author of the preceding Comment provides some counterexamples of non-Lorentzian spectra that have the same spectral entropy. Using such noises, it may be possible to design a protocol with no change of spectral entropy while generating a finite amount of released heat. This would invalidate our proposed relationship of proportionality between entropy and heat.

We agree with the author that this is true. However, in our paper [1], we never claimed that our result is universal regardless of the spectrum. At several points in the paper, we stressed that this result holds for the class of exponentially correlated noises (with Lorentzian spectrum), which depend on a single free parameter, namely, the correlation time τ_c. For other families of spectra, depending on more than one parameter, our result indeed does not hold in a simple and straightforward way. However, this does not mean that one may not find nontrivial extensions to it that are capable of accommodating some spectra other than the Lorentzian ones, for instance, by generalizing the definition of entropy used in our paper [3]. Hence, although our result is not universal, it does pave the way to further investigations of the relationship between the noise correlation and the released heat, which we hope will turn out to be fruitful and inspiring.

The author also points out that, even for Lorentzian spectra, the proportionality relationship between the spectral entropy and the released heat cannot be exact. Both quantities can be computed analytically with the overdamped model used in our work and the resulting mathematical expressions are indeed different. We agree with the author on the mathematical technicalities, and indeed we already performed those calculations in our paper, in Appendixes G and H [1]. In particular, the expression for the released heat is given in Eq. (G9) and the one for the spectral entropy difference in Eq. (H5).

Further, the author tries to reinforce his argument by presenting our results in a slightly different way [Fig. 4(b) of [2]], by plotting the ratio $\Delta Q/\Delta H_S$ and showing that it is not a constant (equal to the equipartition temperature $T_{\rm eq}$).

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However, this representation is not meaningful: By using, as the equipartition temperature, the linear fit to the data of Fig. 4(a), Fig. 4(b) of the preceding Comment [2] simply shows that the fit is not perfect, a rather unsurprising fact. In contrast, in our own work, the equipartition temperature is *measured* experimentally and it is such an experimental value that is used when plotting the relationship between entropy and heat (Fig. 4 in our published paper [1]).

Finally, we would like to stress a general point regarding our work. The two analytical expressions for ΔQ and ΔH_S are indeed mathematically different. They describe two quantities of different physical nature, one being evaluated from the spectrum of an external noise and the other from the motional fluctuations of the microsphere. The strong result of our work (Figs. 4 and 20 of the original paper [1]) lies in the very clear experimental correlation between those two quantities, within the range of parameters considered in our experiments. This is interesting precisely because of their very different nature, a point that the author of the preceding Comment seems to miss. The different mathematical expressions of ΔQ and ΔH_S cannot disprove the observed correlation.

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[1] R. Goerlich, L. B. Pires, G. Manfredi, P.-A. Hervieux, and C. Genet, Harvesting information to control nonequilibrium states of active matter, Phys. Rev. E **106**, 054617 (2022).

[2] A. Bérut, preceding Comment, Comment on "Harvesting information to control nonequilibrium states of active matter", Phys. Rev. E **107**, 056601 (2023).

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